CHAPTER III.

TO ALTER THE VELOCITY RATIO BY GRADUAL CHANGES.

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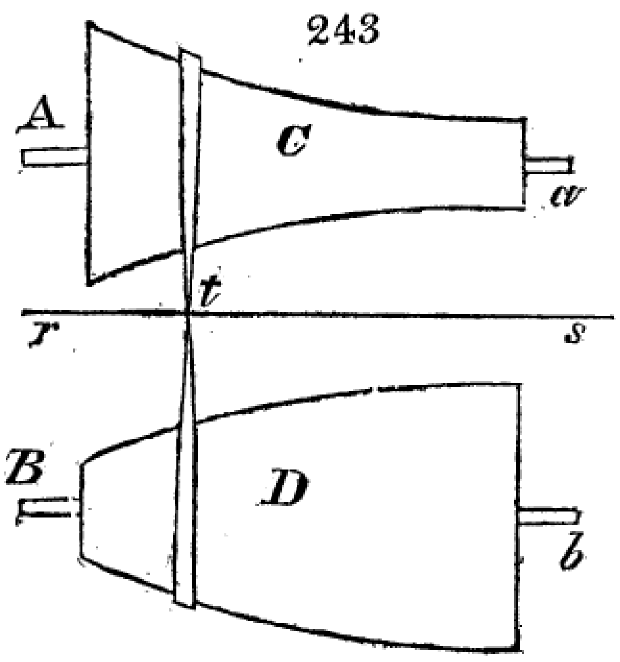
473. IN the methods of the last Chapter it is obviously necessary that the machines should be stopped in order to effect the necessary changes of the wheels, or in the position of the bolts, and so on; and besides, the series of changes themselves are not continuous, and we have only the choice of a few given intermediate ratios between the extremes. We have now to consider how the velocity ratio may be altered by gradual changes, so as to enable us to take any value for it between the extremes. The same constructions will generally enable the changes to be made without interrupting the motions of the machine.

474. Let *Aa*, *Bb*, fig. 243, be parallel axes, *C*, *D*



solids of revolution or long pullies connected by an endless strap. If this strap be crossed and the sum of every opposite pair of diameters of these solids be constant, the strap will be tight in any position upon them. A bar *rs* slides in the direction of its own length, and is provided at *t* with a loop or with friction-rollers, between which the belt passes, and which serves to retain it in its place. In Art. 184 it is shewn that a belt may be guided *by its advancing side* to any point of the surface of a revolving cylinder; and this guide-loop embracing the sides of the belt which are advancing to the two pullies is sufficient to retain them in any position upon their surfaces, provided the tangents to the generating curves of the solids do not make too great an angle with the axis. If the bar were removed, the two ends of the belt would be drawn each towards the large end of its pully, by Art. 181; but the loop is sufficient to prevent this action. By sliding the bar and belt to different points the velocity ratio will be gradually changed as the acting diameters of the driver and follower are thus both gradually altered.

475. The solids are easily formed to suit the condition of the constancy of their added diameters; for draw AM, ab, fig. 244,

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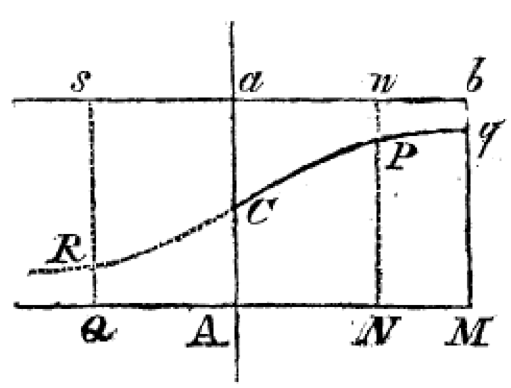

parallel and at a distance equal to the given sum of the radii, and let CPq be the generating curve of one pully round AM, then will the same curve generate the other pully by revolving round ab.

476. Let $AN = x$, $NP = y$, $nP = y_{\prime}$, A and a be angular velocities of the axes AM, ab, respectively,

$$\frac{A}{a} = \frac{y_{\prime}}{y}.$$

Now if the strap is to remain equally tight in every position, we must have $y + y_{,} = c$;

$$\therefore \frac{A}{a} = \frac{c - y}{y}.$$

If the solids be cones, of which $AM = l$, and $Mq = r$,

$$\text{we have } y = \frac{xr}{l}; \therefore \frac{A}{a} = \frac{c - \frac{r}{l} \cdot x}{\frac{r}{l} x} = \frac{\frac{lc}{r} - x}{x}.$$

If equal shifts of the belt between A and M are to produce equal differences in the velocity ratios, we have

$$\frac{A}{a} \propto x \propto \frac{c - y}{y}.$$

If equal shifts of the belt are to produce a geometrical series of velocity ratios, then

$$\frac{NP}{nP}, \text{ or } \frac{y}{c - y} = g^x,$$

and when $x = 0$, $NP = nP$; therefore the origin of x is at the point A, if $AC = aC$,

$$\text{and } \frac{c}{y} = g^{-x} + 1; \therefore y = \frac{c}{g^{-x} + 1}$$

is equation to curve.

$$\text{Also, } c - y = c - \frac{c}{g^{-x} + 1} = \frac{c}{g^x + 1};$$

which shews that if we set off from the point A equal abscissæ AN, AQ, in opposite directions the ordinates NP, sR will be equal.

477. But in practice it is more usual to make the solid pullies into cones, because the strap is apt to slip when the inclination is great. In this case the desired succession

of velocity ratios is obtained by making the shifts of the belt unequal.

When cones are employed,

$$\frac{A}{a} = \frac{\frac{lc}{r} - x}{x}, \text{ and } x = \frac{lc}{r} \times \frac{1}{1 + \frac{A}{a}},$$

from which the shifts or values of x can be computed for any required succession of values in $\frac{A}{a}$.

Sometimes a cone and cylinder are employed for the two solids, but in that case a stretching pully is required for the belt, because the sum of the corresponding diameters is no longer constant. If the cone be the driver the velocity ratio $\frac{a}{A}$ will vary directly as the distance of the belt from the apex of the cone.

478. Variable velocity ratios are also obtained from wrapping connectors by means of pullies so contrived as to expand and contract their acting diameters, the structure of which belonging to constructive mechanism, may be found in Rees' Cyclopædia; they are termed Expanding Riggers.

479. The *disk and roller* is often used for the purpose of obtaining an adjustable velocity ratio by rolling contact.

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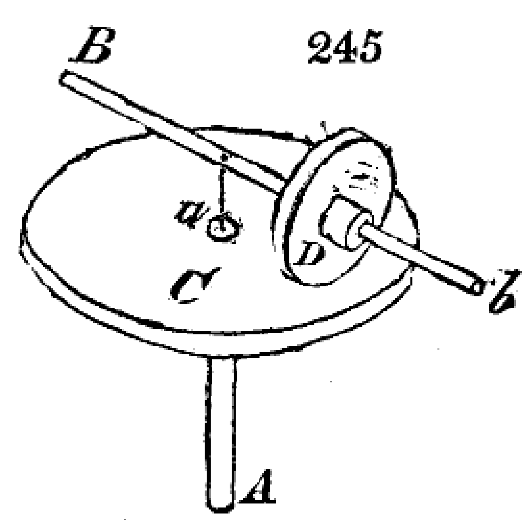

Aa the driving axis, to which is fixed a plain disk *C*. *Bb* the following axis whose direction meets that of *Aa*. A plain roller *D*, whose edge is covered with a narrow belt of soft leather, is mounted upon the axis *Bb*, so that it can be made to slide at pleasure to different distances from the point of intersection of the axes, but yet is prevented from turning with respect to Bb. This roller and its axis will therefore receive from the disk a rotation by rolling contact; and if r be the radius of the roller, R the adjustable radius of its point of contact with the disk, A and a the respective angular velocities of Aa and Bb, we have

$$\frac{a}{A} = \frac{R}{r} \text{ varies directly as } R.$$

But the rolling contact of the surfaces is imperfect, for perfect contact in the case of intersecting axes can only take place between cones whose apex coincides with the point or intersection. The following combination is more perfect in its action, but not so simple in construction.

480. Let AB, fig. 246, be the axis of the driver, which is a solid of revolution whose generating curve is Nn. The

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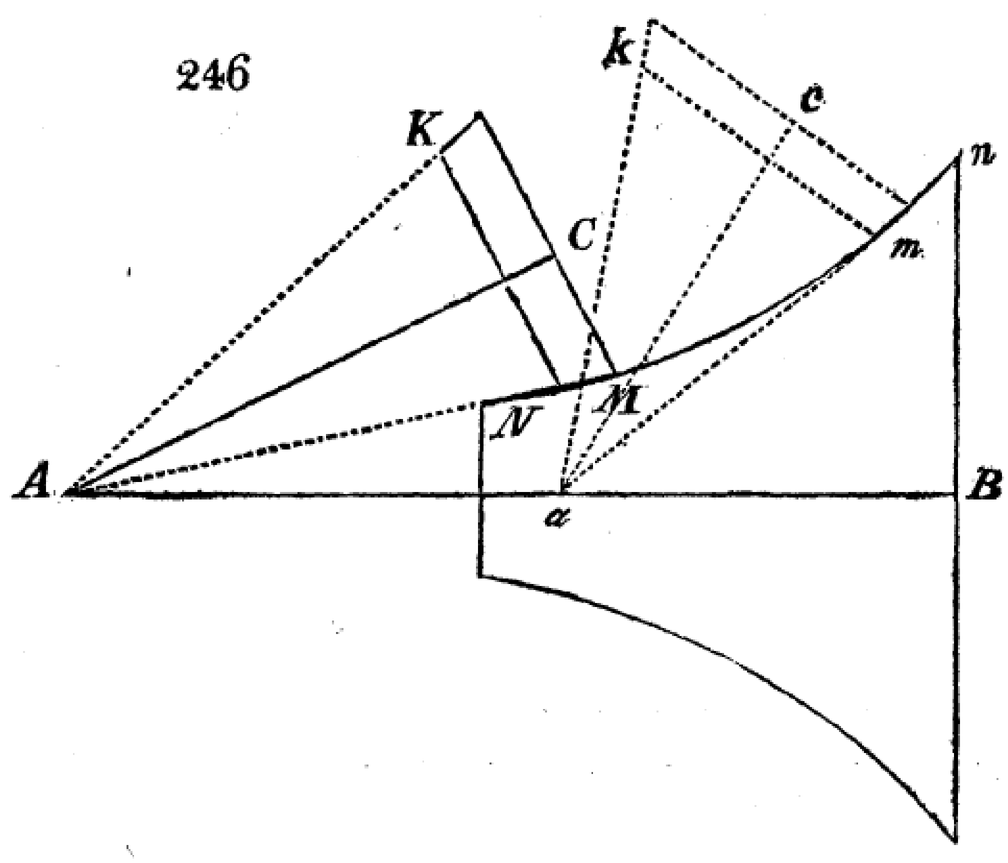

follower is a conical frustum KM, whose axis AC must be mounted in a frame in such a manner that the apex A of the cone may travel in a line Aa coinciding with the axis of the driver, and that the axis AC shall have the power of turning in position about the point A, so as to enable the frustum to rest upon the surface of the solid pully in every position of

AC, and thus to receive motion from it by rolling contact. Thus *km* is a position of the frustum in which it touches the solid at *m*, and its apex has moved from *A* to *a*, still remaining in the line *AaB*. If now the line *AM* touch the generating curve *Nn* in all these positions of *AC*, the portion of the solid in contact with the frustum is so small that it will nearly coincide with the corresponding frustum of a cone whose apex would be at *A*, and therefore coincide with that of the follower. The contact action therefore will in this case be complete.

But *AM* the tangent of *Nn* is thus shewn to be of a constant length, *Nn* is therefore the equitangential curve or tractory (Peacock's Ex. p. 174), to find the equation to which, we have, if *AB* be the axis of x,

$$\tan = \frac{y\sqrt{dx^2 + dy^2}}{dy} = t \text{ a constant.}$$

$$\therefore\ dx = \frac{dy}{y}\sqrt{t^2 - y^2} \text{ is equation to curve;}$$

which integrated gives

$$x = \sqrt{t^2 - y^2} + \frac{t}{2}\log\frac{t - \sqrt{t^2 - y^2}}{t + \sqrt{t^2 - y^2}};$$

whence from assumed values of y the curve may be constructed by points.

$$\sqrt{t^2 - y^2} \text{ is the subtangent} = s \text{ suppose;}$$

$$\therefore\ x = s - \frac{t}{2}\log\frac{t + s}{t - s}.$$

y	s	x
.9	4.72	6.75
1.	4.70	6.29
1.1	4·68	5.80
1.2	4.65	5.29
1.3	4.62	4.88
1.4	4.59	4.53
1.5	4.56	4.23
1.6	4.53	3.97
1.8	4.46	3.47
2.0	4.37	2.97
2.2	4.27	2.54
2.4	4.16	2.17
2.6	4.04	1.85
2.8	3.90	1.54
3	3.76	1.30

In the above table values of y are taken from 3 inches to 9, and the constant tangent $t = 4.8$ inches. From this the curve may be easily constructed by points.

481. The solid cam, (Art. 363) may be used to obtain adjustable motion, in which case the screw a and its nut must be removed, and the cam may then be shifted at pleasure so as to bring any section of it into action upon the follower Dd; and also this section may be allowed to continue its action as long as we please; thus we may, by properly forming the successive sections of the solid, retain at pleasure the law of motion that belongs to any one of them, or gradually change it into that which is appropriated to any other section, by shifting the cam so as to bring that section under the follower.

482. In link-work gradual changes of the velocity ratio are effected by fixing the pins upon the arms in slits or sliding pieces, that thus allow of gradual changes in the effective lengths of these arms upon which the velocity ratio depends. This may be managed in various ways. I shall conclude this Part with a piece of link-work by which such changes may be effected without the use of these adjustable pins.

483. *A*, fig. 247, is the fixed center of motion of a crank or excentric *Am*, which by means of a link *mb* communicates in the usual way a small reciprocating motion to the arm *Bb*, whose center of motion is *B*. The end of *b* is also joined by a link *bc* to an arm *Cc*, therefore the reciprocation of *b* is communicated to *c*. But the center of motion of the arm *B* is itself mounted on a shifting arm whose center is near *b*; and the radial distance of *B* from this center is made equal to the link *bc*; thus the position of the center *B* can be shifted to any point of the arc *Bc*, or even be brought to coincide with *c*. In all these different positions the quantity of motion which *b* receives from *Am* will be nearly the same, but the arc described by *c* will vary; for when the center is at *B*, it will give to *c* very nearly its own motion; but when *B* is moved to *c* it will communicate no motion at all to *c*, for the link *bc* will then coincide with *Bb*, and will vibrate as one piece with it round the point *c*. In any intermediate positions of *Bb*, as at *Db*, the velocity of *c* and the extent of its excursion will vary nearly as *Dn*, the perpendicular upon *bc*, which vanishes when *D* comes to *c*.

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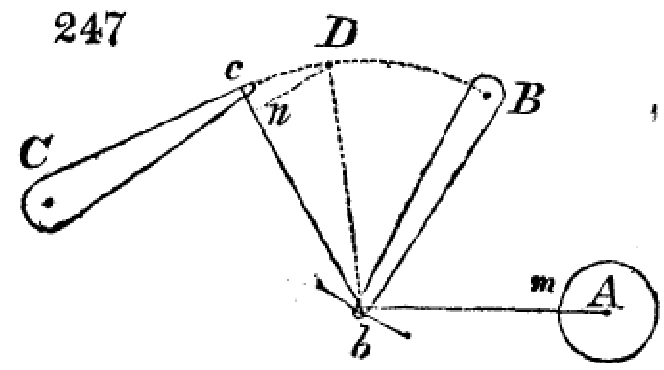

As the travelling of the center *B* does not stop the motion of the system, this combination affords a ready method of adjusting the relative velocity in link-work, or of entirely cutting off the motion of the follower *Cc* without stopping the motion of the driver *Am*.

THE END.