PART THE SECOND.

ON AGGREGATE COMBINATIONS.

CHAPTER I.

GENERAL PRINCIPLES OF AGGREGATE MOTION.

386. The motion of a point with respect either to its path or velocity may be considered as the resultant of two or more component motions. If it happen that the latter taken separately are more simple and more easily communicated than the resultant motion, it is evident that this may be advantageously obtained by communicating simultaneously to the given point the component motions. For an example of an aggregate path, let it be required to make a point describe an epicycloid. Every epicycloidal path may be resolved into two circular paths, one of which represents the base of the epicycloid, and the other the describing circle. And if the point be attached to a disk or arm which revolves uniformly round its own center, while at the same time that center revolves uniformly round the center of the base in a plane parallel to that of the first revolution, the point will describe an epicycloid, the nature and proportions of which will depend upon the proportion of the radii of the two circular component paths, and upon the relative time and directions of their revolutions. In this example a very complex path is referred to two paths of the simplest nature, and the question is one case of a general problem that may be thus enunciated:—*To cause a point to move*

in a required path by communicating to it simultaneously two or more motions in space.

387. As an example of motion complex in velocity, but simple with respect to its path, let a body be required to travel in a right line by a reciprocating motion, but always making its forward trip through a space greater than its backward trip, and thereby gradually advancing from one end of the path to the other. This motion may be resolved into a reciprocating motion of equal advance and retreat, combined with a simple slow forward motion.

If therefore the body be mounted on a carriage or frame which advances slowly in the required direction, and if at the same time a common reciprocating motion be given to the body with respect to the carriage; the question will be answered by referring the given compound motion to two of a simple and practicable nature.

388. Again, let a body be required to move so very slowly in a right line, that in the ordinary methods a long train of wheel-work or of other combinations would be required to reduce sufficiently the velocity of the original driver. But if this small velocity be considered as the difference of two velocities in opposite directions, then it may be obtained by mounting as before the body on a carriage which proceeds with any convenient velocity in one direction, while the body moves with respect to the carriage with a nearly equal velocity in the opposite direction.

These examples belong to a second problem which may be thus stated:—*To produce the motion of a piece in a given path by communicating to it simultaneously two or more motions in that path, either in the same or in opposite directions.*

389. In these examples, however, it appears that the frame or part of the machine which determines the path of

one of the component motions is itself in motion. In the first example, the center of motion of the revolving piece which carries the describing point itself travels in a circle; and in the second example, the slide upon which the point that receives the aggregate motion is made to move, is itself also in motion. And this, from the nature of Aggregate Combinations, will always be the case; and as these bodies which travel in moving paths have to derive their motion from a driver whose path is in the usual manner stationary, it appears that to carry this aggregate principle into effect, requires that we should have the means of communicating motion from a driver to a follower, when the respective position of their paths is variable.

I shall therefore begin by giving examples of the methods by which this may be effected.

To connect a Driver and Follower, the relative position of whose paths is variable.

390. If the center of motion of a toothed wheel itself travel in a circle parallel to the plane of rotation, then a second wheel concentric with the circular path, and in geer with the travelling wheel will remain in geer with it in all positions of its center; or if the center of the wheel travel in a right line parallel to the plane of rotation, a rack parallel to its path will always remain in geer with the wheel, and communicate a motion to it; as will also an endless screw, as in fig. 198, where *Aa* is a long endless screw, *B* the travelling wheel whose center of motion moves in the path *Bb*, parallel to the axis of the screw. The screw

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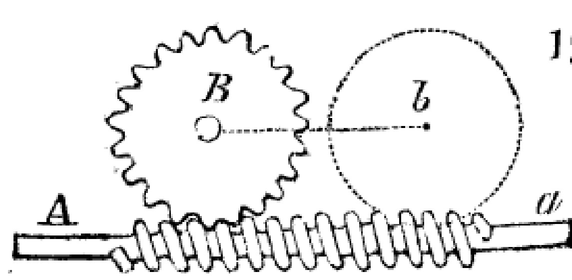
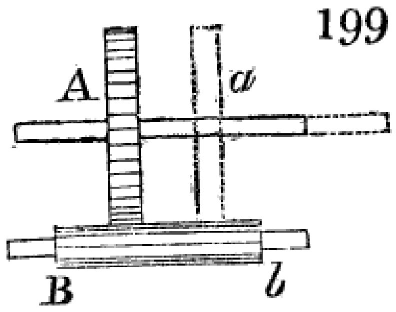


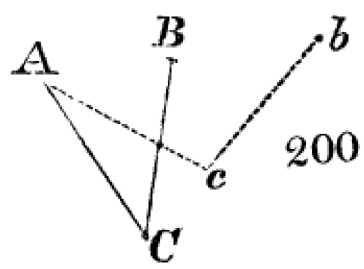
will therefore act upon the wheel whatever be the position of its center upon this line, and will also allow the center to be moved into any position upon the surface of the cylinder that would be generated by the motion of *Bb* round *Aa*, the plane of the wheel of course always passing through the axis *Aa*.

Again, if the wheel be required to travel in the direction of its own axis, as from *A* to *a*, fig. 199, a *long pinion Bb* will retain its action upon it in all its positions.



199

But if the center of the wheel is to travel in any other curve in a plane perpendicular to its axis, let *A*, fig. 200, be a fixed center of motion, *B* the travelling center of motion, and let *AC*, *CB* be a frame jointed at *C*; then if *B* be moved into any position within the circle whose radius is *AC* + *CB*, the frame will follow it, the angle *ACB* becoming greater or less according to the radial distance of *B* from *A*. Let a center of motion be placed at *C*, then will three wheels whose centers are *A*, *C*, and *B*, remain in geer in all these positions of the frame, and thus allow *B* to travel in any curve without losing its connexion with the central wheel at *A*.



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391. The same principles also apply to centers of motion connected by sliding contact or wrapping connectors; for generally, it is evident, that if two parallel axes be connected by any of the contrivances for communicating unlimited rotation, one axis may travel round the other in the circle whose radius is the perpendicular distance of the axes, without disturbing their connexion. Other expedients are also employed, which belong rather to constructive mechanism. Thus, instead of the long pinion *Bb*, fig. 199, a

short pinion may be used which can slide along its axis, but not turn with respect to it, and this pinion may be made to follow the wheel *A* in its motions. But, in fact, as we advance in our subject, the combinations necessarily increase in number and complexity under each head to such a degree, that it becomes impossible to include them all in the limited space of such a treatise as this. I shall therefore merely give examples of one or two of the least obvious arrangements; others will occur during the calculations of Aggregate Motion in the succeeding Chapters.

392. A travelling pully which derives its rotation from another pully with a fixed axis of motion, may have its own axis carried about to any relative position with the first, provided the wrapping band have a suspended stretching pully to keep it tight in all these changes of distance, and that the pully travel only in its own plane, and consequently its axis always remains parallel to that of the other pully. For if it move out of that plane the wrapping band will be thrown off the pully (Art. 184). Fig 201 is one arrangement by which the pully may be also allowed to move in the direction of its axis*.

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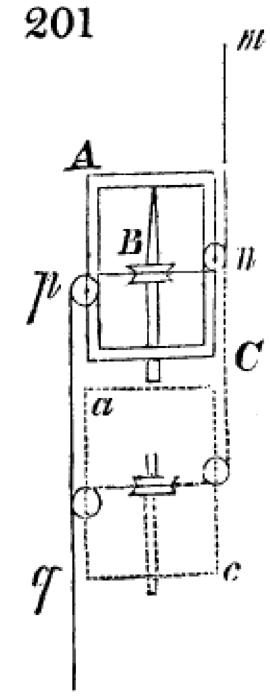


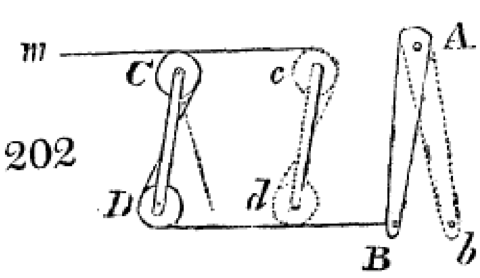
B is the pully whose axis is mounted in a frame *AC*, to whose sides are fixed the axes of guide-pullies *n*, *p*; the wrapping band is passed over these pullies as at *mnpq*, making one turn round the pully *B* in its passage; the ends *mn*, *pq* of the band are carried parallel to the axis of *B*, and passed over proper guide-pullies to the driving wheel. The frame *AC* may evidently be moved into any other position *ac*, in the plane *mq*, without disturbing either the tension of the band or its connexion with *B*.

* Lanz and Betancourt (Anal. Essay, D. 20.) have a somewhat similar arrangement.

393. Two arms *AP*, *CD* (fig. 114, p. 190), being connected by a link *PD*, the center of motion *C* of one of them may be shifted into various positions with respect to *A*, without breaking the connexion of the system; but the velocity ratio of the arms will necessarily be different in every new position. If the arms have only a small angular motion, as in the Article referred to, the center *C* may receive a small travelling motion in a direction perpendicular to *PD*, without materially altering the velocity ratio.

Fig. 202 is an expedient by which this communication can be maintained between shifting centers without affecting the velocity ratio.

202



AB is the arm whose center of motion *A* is fixed, *CD* the arm whose center of motion travels in the line *Cc*; guide-pullies *C*, *D* are mounted, one concentric to *C*, and the other at the extremity *D* of the arm. A line is fixed at *m*, passed over the pullies *C* and *D*, and attached to *B*. If *B* be moved to *b* it will, by means of this line, communicate the same motion to *CD* round *C* as if it were a link jointed in the usual way at *D* and *B*. But the peculiar arrangement of the line allows the center of the arm to be removed to any other point in *Cc*, as to *c*, without interrupting the connexion of *B* with its extremity. The arm is supposed to be returned by a spring or weight.