

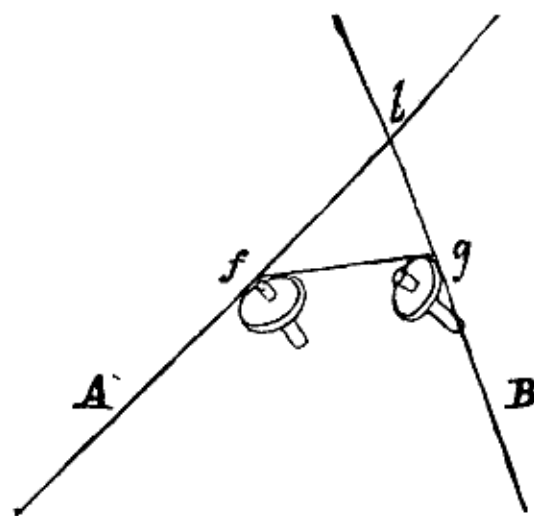
CHAPTER VI.  
ELEMENTARY COMBINATIONS.

CLASS A.  $\left\{ \begin{array}{l} \text{DIRECTIONAL RELATION CONSTANT.} \\ \text{VELOCITY RATIO CONSTANT.} \end{array} \right.$

DIVISION E. COMMUNICATION OF MOTION BY REDUPLICATION.

208. THE mechanism which results from this principle forms a class which is already separated by common practice from all others under the name of *Tackle*, and is principally employed on shore for raising weights, but in the rigging of ships is used to give motion to the sails, in order either to place them in the requisite positions for receiving the action of the wind, or to furl and unfurl them.

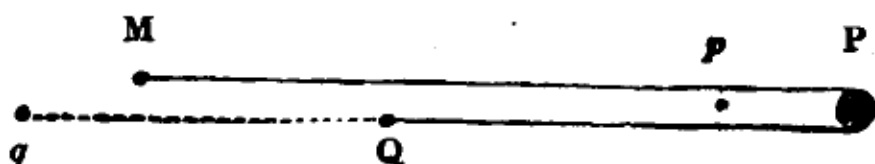
209. If an inextensible string  $AfgB$  be passed over any number of fixed pins, as  $f$  and  $g$ , and if the extremities  $A, B$  of the string be compelled to move each in the direction of its own portion,  $Af, gB$  of the string, then the motion of one of these extremities will evidently be communicated unaltered to the other,



and every intermediate portion of the string will move with the same velocity. This is unaffected by the form of the pins over which the string passes, and they may therefore be fixed cylinders or pulleys, that is to say, wheels mounted on revolving axes, which are generally substituted for fixed pins, for the purpose of reducing the friction of the string in passing over them.

210. If, however, the pins or axes of the pulleys be not fixed, then the principle of *reduplication* (Art. 30) is introduced, by which the velocity of the string and its extremities is greatly modified.

Thus let the string be attached to a fixed point  $M$ , and then doubled over  $P$ , and returned to  $Q$ ,  $PQ$  being parallel



to  $MP$ ; also let  $P$  be capable of moving in a path parallel to  $MP$ , then if  $Q$  be moved to  $q$ ,  $P$  will travel to  $p$ ; and it has been shewn in Art. 30, that  $qQ = 2pP$ . Also, the portion of string  $Mp$  is at rest while every point of  $PQ$  travels with a velocity equal to that of its extremity  $Q$ .

Now let the string be wound back and forwards, beginning with the fixed extremity at  $M$ , and passing alternately over  $P$  and  $M$ , finally ending at  $Q$ ; and let the number of strings at  $P$  be  $n$ , which will manifestly be an even number; then if a motion be communicated to  $Q$ , which carries it to  $q$ ,  $P$  will be moved to  $p$ ; and as the string is inextensible, its total length in both positions will be the same; that is,

$$\overline{n - 1} \cdot MP + PQ = \overline{n - 1} \cdot Mp + pq,$$

$$\text{or } \overline{n - 1} (Mp + pP) + Pp + pQ = \overline{n - 1} Mp + pQ + Qq;$$

$$\therefore n \cdot Pp = Qq.$$

If the extremity  $Q$  were once more passed over the pin  $M$ , and carried into any convenient direction, the velocity of its extremity along that direction would be plainly unaltered.

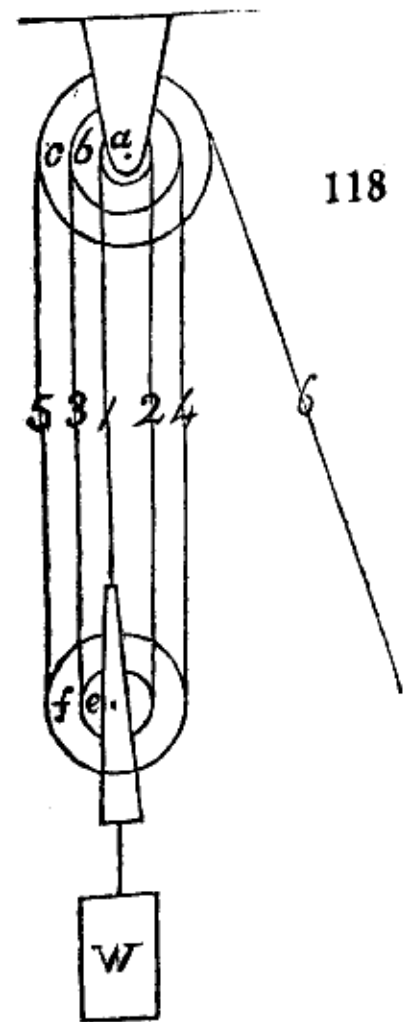
But if the end of the string were first tied to the moveable pin  $P$ , and then wound in the same manner back and forwards over the two pins  $P$  and  $M$ , finally ending at  $Q$ , then the number  $n$  of strings at the moveable pin would be an odd number, and we should find in the same manner,  $n \cdot Pp = Qq$ , also the velocity of the extremity  $Q$  would be unal-

tered by passing it at last over the fixed pin, and carrying it from that in any convenient direction.

211. In practice these fixed and moveable pins are replaced by *blocks*, each of which contains as many mortises as the reduplication of the string requires, and in each mortise is a friction-pully or *sheave*\*, having a groove in its circumference round which the string or cord passes. The entire assemblage, consisting of a fixed and moveable block with the cord, is termed a *Tackle*†. The pulleys may be arranged in various ways in the block, which are represented in the ordinary treatises on mechanics. As however the diameter of the pulleys has been shewn to produce no effect upon the velocity ratios of the combination, it is most convenient to represent the sheaves as in fig. 118, where they are shewn as concentric, but of different diameters, and for the purpose of exhibiting the course of the string with more clearness.

212. In this figure the string is attached to the lower or moveable block, and as there are five strings at this block, we have  $n = 5$ , and the velocity of the extremity 6  $\dot{=} 5 \times$  velocity of  $W$ , by Art. 210.

The upper pully being fixed, it is plain that the strings 1 & 2, 3 & 4, 5 & 6, move respectively with the same velocity, but in opposite directions, 1, 3, and 5 ascending, and 2, 4, 6 descending, if  $W$  be supposed to move upward, and *vice versa*. Also the velocities of each of these pairs of



\* From *Scheibe*. Germ.

† This term appears to have been derived thus: τροχαλία, Gr.; *Trochlea*, Lat.; *Taglia*, Ital.; *Taakel*, Dutch. In French, *Mouffle* is used either for the block alone, or for the block and its sheaves; and *Pully* (Eng.), as well as *Poulie* (Fr.), is used either for the sheave or for the complete block and its sheaves.

the string are different, for the velocity of 1 is equal to that of the lower block; and if 3 were the extremity of the string, 1, 2, 3 would with their sheaves form a tackle in which  $n = 3$ ; and therefore the velocity of 3 is triple that of the lower block; similarly, the strings 1 to 5 form a tackle in which  $n = 5$ ; and thus, whatever odd number of strings are at the lower block, the velocities of these strings, beginning from the center, will be 1, 2, 3, ..... an arithmetical series of the odd numbers, in which the velocity of  $W$  is supposed unity; but if one end of the string be tied to the fixed block, and consequently the number of strings at the moveable block be even, then the series of velocities can be similarly shewn to be 0, 2, 4, 6, .....

213. In figure 118 the sheaves  $a, b, c, \dots$  are supposed to revolve separately, although upon the same axis; but since the perimetral velocity of a wheel varies directly as the radius, and the strings of the tackle have been shewn to move with velocities increasing in an arithmetical progression, it follows that if the lengths of the radii of the sheaves  $a, b, c, \dots$  form the same progression as the velocities of the strings, the sheaves will all revolve with the same angular velocity, and may consequently be all made in one piece. Blocks so fitted up form what is termed White's Tackle, from the name of its inventor.\*

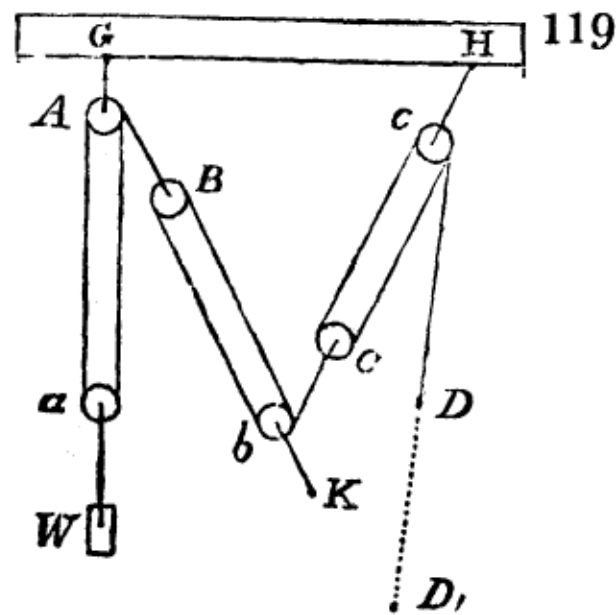
214. The free portion of rope (as 6) is termed the *fall*, and when the other extremity (1) is tied to the fixed block, and therefore, as we have seen, has no velocity; this is termed the *standing part*. In nautical phraseology, the following terms are applied to *Tackles*. If  $n = 1$ , the tackle is a *Whip*; if  $n = 2$ , it is a *Gun-Tackle*; if  $n = 3$ , a *Luff-Tackle*; the fall being in all cases supposed to be taken

\* White's Century of Inventions, p. 33.

from the fixed block. It may however be observed, that in any given tackle the velocity ratio is different according as one block or the other is made the fixed block. Thus in fig. 118 the block from which the *fall* proceeds is made the fixed block, and  $n = 5$ ; but if this block were employed as the moveable block, we should have  $n = 6$ . The number of sheaves is always less by one than the number of strings at the *fall-block*.

The fall-block is usually fixed, because this allows the fall-rope to be drawn in any direction, whereas if the fall proceed from the moveable block, it must be drawn as nearly as possible in a direction parallel to the path of the moving body, and therefore to the strings of the tackle\*.

215. Several tackles may be combined, as shewn in fig. 119. Thus let  $A$  be the fixed block,  $a$  the moveable



block of a tackle in which there are  $n_1$  strings at  $a$ , and of which  $AB$  is the fall; let the extremity of this fall be tied to the moveable block  $B$  of a second tackle of which  $b$  is the fixed block, and  $n_2$  the number of strings at  $B$ . Also, let the fall  $bc$  of the second tackle be tied to the moveable block  $C$  of a third tackle of which  $c$  is the fixed block, and  $cD$  the fall, and  $n_3$  the strings at  $C$ ; let a velocity  $V_1$  be given to

\* Vide Reduplication, in Chap. VIII.

$D$ , and let  $V_1, V_2, V_3$ , be the velocities of  $W, B$  and  $C$  respectively ;

$$\text{then } V_4 = n_3 V_3 = n_3 n_2 V_2 = n_3 n_2 n_1 V_1.$$

If there be  $m$  tackles in this series or *train*, and they have all the same number of strings, we should find in a similar way  $V_{m+1} = n \cdot^m V_1$ .

Now the total number of strings in this combination =  $n \times m$  ; whence the following problem.

216. Given the velocity ratio  $\frac{V_{m+1}}{V_1} = n^m$  of the train of tackles, to find the number and nature of the separate tackles that will require the fewest strings.

Here  $n^m = \text{constant} = C$  suppose ;

$$\begin{aligned} \therefore m &= \frac{1 C}{1 n} \text{ and the number of strings} \\ &= m n = \frac{n \cdot 1 C}{1 n}, \end{aligned}$$

which is at a minimum when  $\text{hyp. log } n = 1$ , and  $n = 2.72$  ; the nearest whole number to which being 3, it appears that a series of Luff-tackles will produce a given velocity ratio with fewer strings than any single tackle or combination of equal tackles. In fact, sailors combine two Luff-tackles in this manner, which they term *Luff upon Luff*.

If however instead of attaching each tackle to a fall from the fixed block of the previous one, it be tied to a fall from the moveable block, one sheave will be saved out of each tackle without altering the velocity ratio, and the total number of sheaves will be  $(n - 1) \cdot m$  ; which will be at a minimum when  $n - 1 = 2.72$ , and  $\therefore n = 3.72$ .

A combination of this kind in which  $n = 2$ , and therefore each pulley hangs by a separate string, is commonly represented in mechanical treatises.