

CHAPTER V.
ELEMENTARY COMBINATIONS.

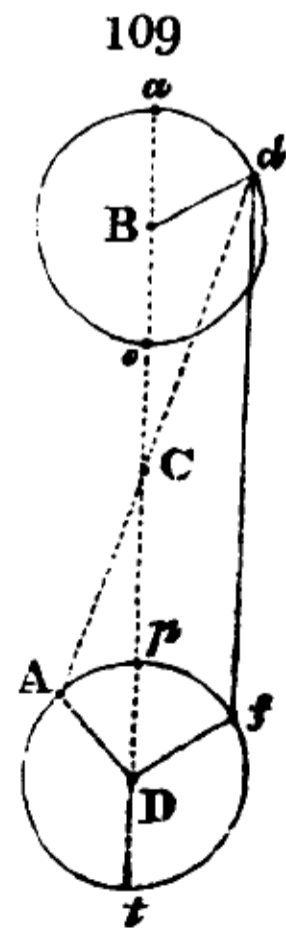
CLASS A. { DIRECTIONAL RELATION CONSTANT.
VELOCITY RATIO CONSTANT.

DIVISION D. COMMUNICATION OF MOTION BY LINK-
WORK.

195. WHEN two arms revolving in the same plane are connected by a link (Art. 32), their angular velocities are inversely as the segments into which the link divides the line of centers. This relation is constantly changing, as the arms revolve, unless the point of intersection T (fig. 6), be thrown to an infinite distance, by making PQ parallel to AB , in all positions, which can only be effected by making the arms equal, and the link equal in length to the distance between the centers. In this case the angular velocities will become equal, and their ratio consequently constant.

196. This produces the arrangement of fig. 109. D, B are centers of motion, $Bd = Df$ the arms, df ($= BD$) the link. If Bd be carried round the circle, $BdfD$ will always be a parallelogram, and consequently the angular distances of Bd and Df from the line of centers the same, and their angular velocity the same.

But in any given position of one of the arms Bd , there are two possible corresponding positions of the arm Df , for with center d , and radius df , describe an arc which will necessarily cut the circular path of f round D in two points f and A ; therefore AD is also



a position of the arm corresponding to Bd , in which the link dA intersects the line of centers in a point C ; and if Bd be moved, the point C will shift its place, and consequently the angular velocity of AD will not preserve a constant ratio to that of Bd .

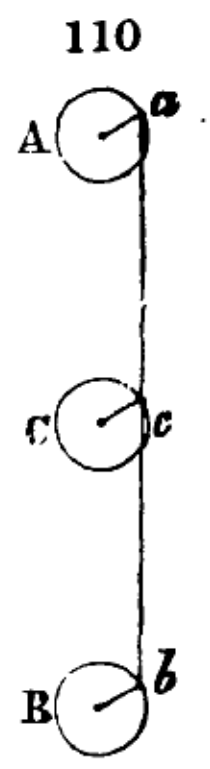
It appears, then, that this system is capable of two arrangements, one in which the angular velocity ratio is constant, and the other in which it is variable, according as the link is placed parallel to the line of centers, or across it.

But if the motion of this system in either state be followed round the circle, it will be found that when the extremity d of the arm Bd comes to the line of centers, either above or below, at a or s , the extremity of the other arm will also coincide with that line, since the link is equal to BD , and therefore to ap or st . In these two phases (Art. 17) of its motion the two positions fd , Ad of the link coincide, and at starting from either of these phases, the link has the choice of the two positions. If, for example, the arms be at Ba and Dp , then as a moves towards d , p may either move towards f , in which case the link will remain parallel to BD , until the semicircle is completed, or else p may move towards A , and then the link will lie across BD , until the semicircle is completed by d coming to s , when a new choice is possible. But in any given position of Bd intermediate between Ba and Bs , it is impossible to shift the link from one position to the other without bending it.

The two phases in which the arms coincide with the line of centers, are termed the *dead points* of the system.

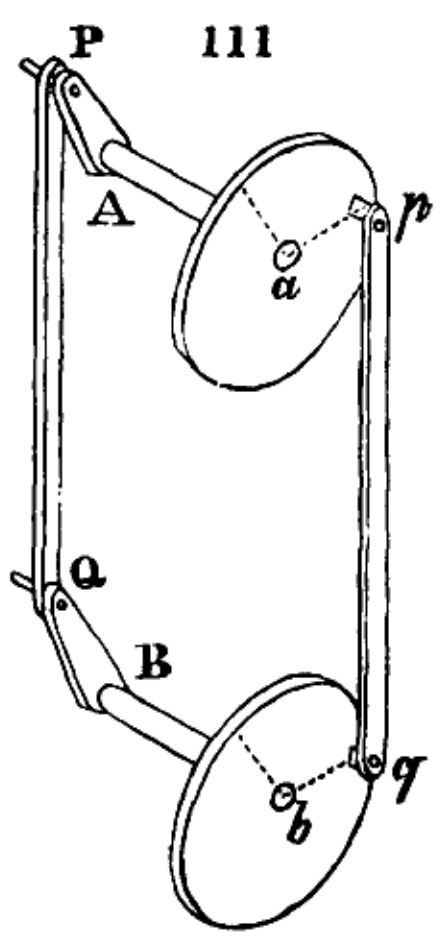
197. When this contrivance is employed to communicate a constant velocity ratio, some provision must be made to prevent the link from shifting out of the parallel position into the cross position, when the arms reach the dead points.

There are three ways of passing the link parallel to itself across the line of centers. First, by introducing a third arm, as at *c*, of the same length as the others, with its center placed on the line of centers, and its extremity jointed to the link, so as to divide the latter in the same proportion as the line of centers is divided by the center of the new arm. This new arm may be placed either between or beyond the others, and plainly renders any position of the link, except that of parallelism to the line of centers, impossible. It is not even necessary that the centers of the three equal arms shall lie in one line, for if the three joint-holes, *a*, *b*, *c*, of the link, be the points of an equal and similarly placed triangle to that formed by the three centers of motion, the arms will all revolve alike.



198. The second way requires only two axes of motion, but has two sets of arms.

Aa, *Bb*, fig. 111, are the two parallel axes. At one end of each are fixed the equal arms *AP*, *BQ*, connected as before by a link $PQ = AB$; at the other end of each are fixed arms *ap*, *bq*, also connected by a link, $pq = ab$.

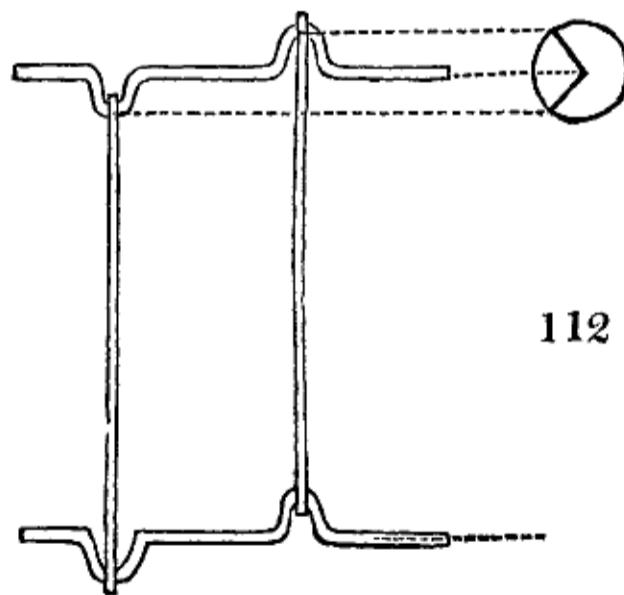


Now since the separate effect of each of these systems is to produce equal rotation in the axes, it is plain that the action of the second will con-
spire with that of the first to produce this effect, whatever be the angle which *AP* makes with *ap*. Let *ap* then be set at right angles, or nearly so, to *AP*; therefore when either system arrives at the dead points, the other will be half way between them, and by communicating at that moment the equal rotation to

the axes, will thus carry the link of the former system over the dead points, without allowing it the choice of the second set of positions; which second set of positions is besides rendered geometrically impossible by this combination of the two sets of arms.

199. The form of the piece to which the joint-pin is fixed is indifferent; thus (fig. 111) the pin P is carried by an arm AP , and the pin p by a disk; but the motion produced by each is precisely the same; the effective length of the arm being in every case measured in the plane of rotation in a right line from the center of the pin to the center of motion of the piece which carries it, whatever be the form given to the latter.

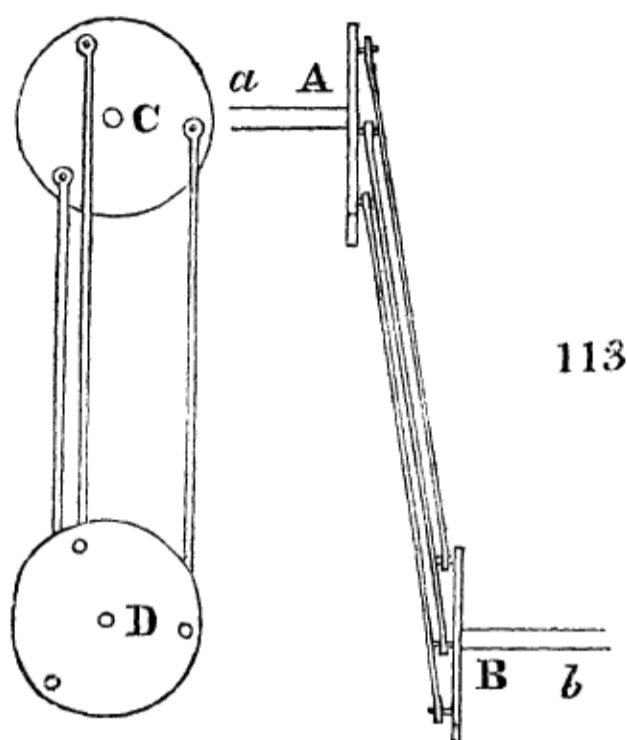
However, if either axis be carried across the plane of motion of the link, the latter will strike against it, and thus prevent the completion of a single revolution. If the axes be required to revolve continually in the same direction, either the piece which carries the pin must be fixed to the extremity of the axis, as in fig. 111, or else the axis must be bent into a loop or *crank*, as it is termed, as in fig. 112, by



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which the axis is also removed from the plane of rotation of the link; but the axis may thus be extended indefinitely on either side.

200. The third method of passing the links over the dead points consists, like the latter, in employing two or more sets of arms and links, so disposed as that only one set shall be passing the dead point at the same moment. But in this method, fig. 113, the axes Aa , Bb are parallel but not oppo-



site, and a disk of any convenient form, as C , D , being attached to the free end of each, pins are fixed in the faces of the disks at equal distances from the centers of motion, and at equal angular distances from each other respectively, and links each equal to the distance of the centers are jointed to them in order, as shewn in the figure.

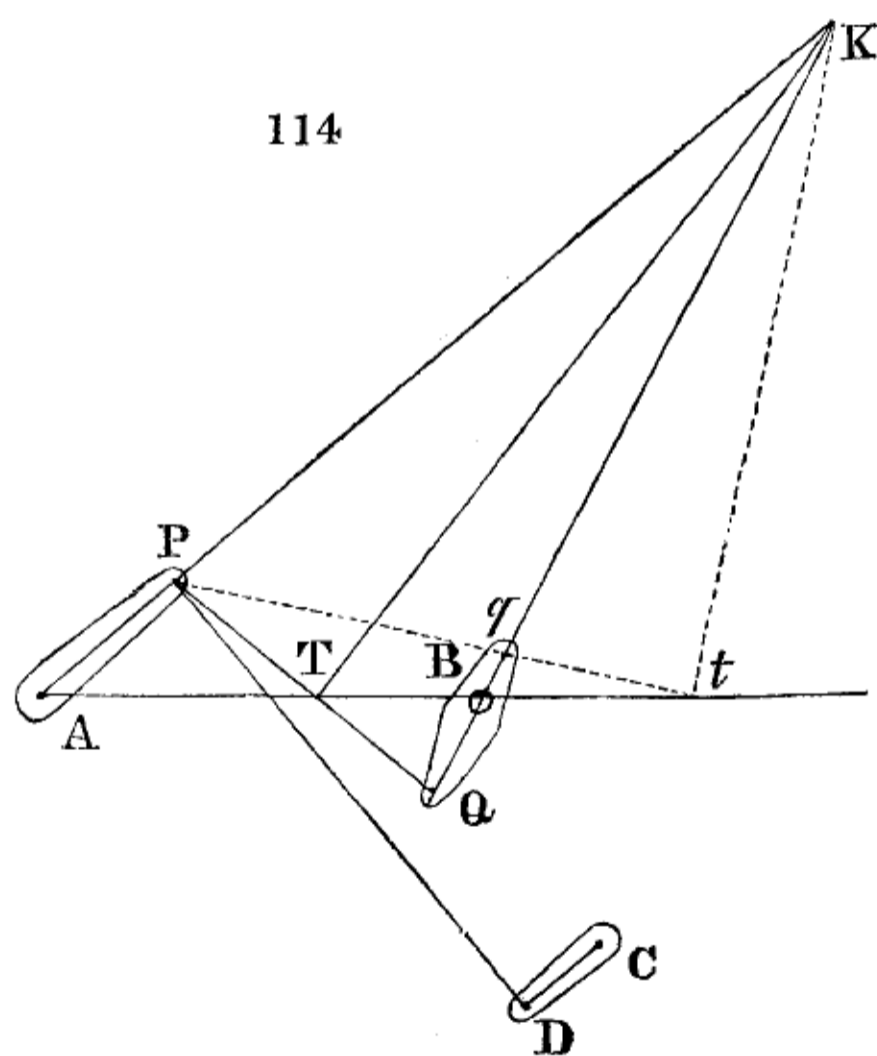
The planes of rotation of these disks are removed from each other by a distance sufficient to throw the connecting links into a slightly oblique position, which enables them each to clear the others, during the rotation, by passing alternately above and below them.

The number of the links is indifferent. Two are sufficient, as in the former case, and the radii of their pins must be nearly at right angles; but if three or more be employed, the pins may be at equal angular distances round the circle; and it is hardly necessary to add, that in determining the length of the links allowance must be made for the oblique

position into which they are thrown by the nature of the contrivance*.

201. It appears (Art. 195), that by Link-work, rotation in a constant velocity ratio can only be communicated between two axes when they are parallel, move in the same direction, and revolve in equal times. If, however, only a motion through a small angle is required, it may be communicated with an approximately constant velocity ratio, whatever be the magnitude of that ratio, the relative position of the axes, or the directional relation.

For if the axes be parallel, it is shewn in Art. 133, that if a pair of arms AP , BQ , fig. 114, be connected by a link



PQ , and placed in such a position that the intersection T of the link and line of centers shall coincide with the perpendicular KT upon the link from the intersection of the arms produced, then will the angular velocity be momentarily constant, and will be sufficiently near to constancy, if

* By T. Bœhm, of Bavaria, communicated to Soc. Arts. vol. I. p. 83.

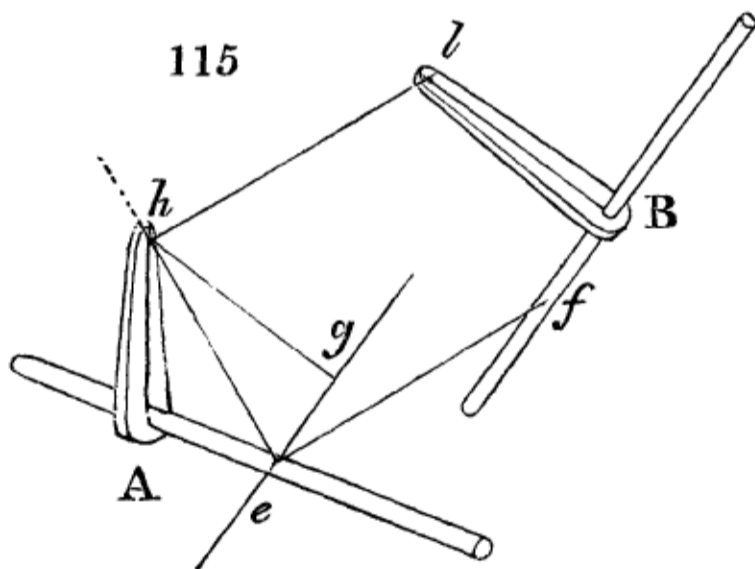
the motion of the links be confined to a small angle on each side of the mean position.

Now the arms AP , BQ will revolve in opposite directions; but if they be required to revolve in the same direction, the centers of motion must lie on the same side of the link. AP , Bq , are a pair of arms connected by a link Pq , which will fulfil this latter condition, and Kt the corresponding perpendicular upon the link produced, and intersecting it in t in the line of centers produced.

The angular velocities of the arms have been shewn to be inversely as the segments AT , BT , or At , Bt .

The simplest mode of arranging the proportions is to make the link perpendicular to the arms in the mean position, as shewn in AP , CD ; PD being the link; and in this case, the angular velocities are inversely as the length of the arms themselves, (Art. 137).

202. If the axes be not parallel, let Ae , Bf (fig. 115), be the axes whose directions do not meet, find their common



perpendicular ef , and draw eg parallel to fB . In the plane Aeg draw eh dividing the angle Aeg into two, Aeh , heg ; whose sines are inversely as the angular velocities of the axes Ae , Bf respectively (Art. 44). From any point h drop perpendiculars hA , hg , upon Ae and eg ; make fB equal to eg , draw Bf equal and parallel to gh , and join hl ; which

being parallel to ef , is plainly perpendicular both to Ah and to Bl .

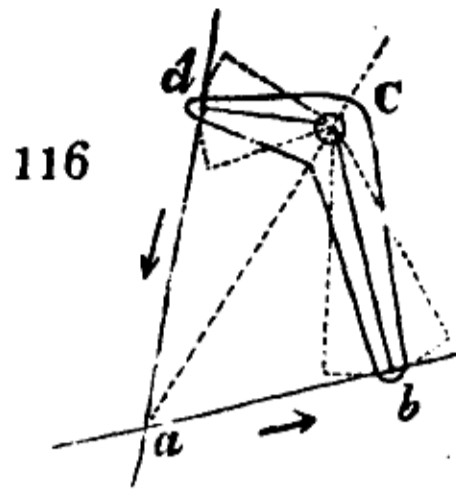
If Ah , Bl be arms, and hl the link, then by the construction the link is perpendicular to the arms; and if the angular motion be small and the figure represent the mean position, the angular velocity ratio of the axes will not differ sensibly from that which would be communicated if the axes were parallel, and the arms and link in one plane, and will therefore be nearly constant, and equal to the inverse ratio of the length of the arms.

If the axes be required to revolve with the opposite directional relation to that shewn in the figure, one of the arms must be placed on the opposite side of the axis. In fact, as each arm admits of two positions (thus h may be above the axis or below it), so there are four ways in which these arms may be combined, two of which will make the axes revolve one way with respect to each other, and the other two the opposite way.

203. The mechanism of Organs, Pedal-Harps, Bell-hanging, and various other portions of machinery, generally called *bell-crank work*, fall under this class of small sensibly equable angular motions. The same kind of mechanism requires the change of the line of direction of these small motions. This may generally be effected by a single axis with two arms; and by the same combination the velocities may be changed in any required ratio, whether the motions be in the same or in different planes, as follows.

204. If the motions be in one plane, let ab , da (fig. 116) be the lines of direction of the motions meeting in a . Draw Ca dividing the angle bad into two, whose sines are in the ratio of the given velocities in ab , da (vide the construction in Art. 44). In aC take any convenient point C for

a center of motion, from which drop perpendiculars Cb , Cd upon the respective directions. If these be taken for arms moving round C , and links be jointed to them in the lines of direction ab , da , then a small motion given to ab will turn the two-armed piece bCd round its axis C , but will not remove its extremities sensibly from the directions ab , da , which are the tangents to the circles described by those extremities in the mean position of the axes. But these extremities will move with velocities which are directly as the length of the arms. (Art. 11).



In practice it is better to make the lines ab and ad bisect the versines of the arcs of excursion, in which case each link will be carried to the right and left of its mean position, instead of deviating wholly towards the center of motion, as in the figure.

205. Since the arcs of excursion of the extremities d , b are given, we can by removing the center C to a sufficient distance from a , reduce the angular motion of the piece as much as we please, and thereby diminish the deviations of a , b from the mean positions*.

A two-armed piece or bent lever of this kind is termed a *crank*, or more properly a *bell-crank*, to distinguish it from the looped axis to which the term crank is also applied (Art. 199), but which differs from it considerably; the object of the

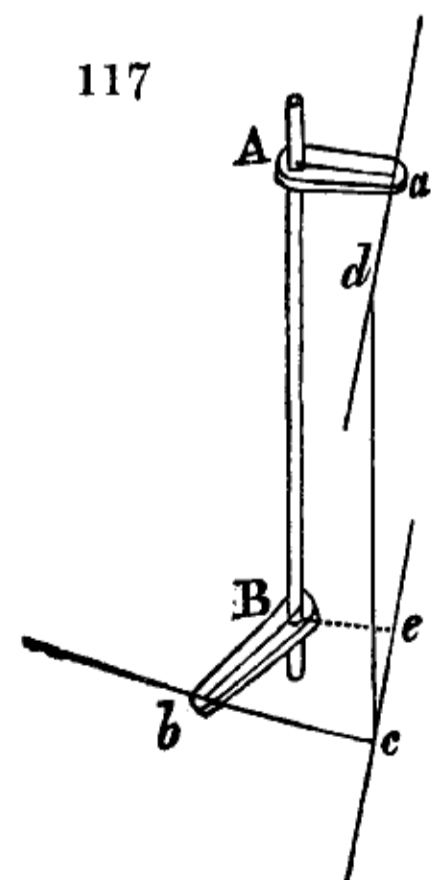
* If the links be not perpendicular to the arms in the mean position, but if the angle adC made by one link with its arm be equal to the supplement of the angle abC made by the other link with its arm, then it can be shewn that during a small angular motion of the system the ratio of the velocities of the links will still remain constant, and be equal to the ratio of the respective perpendiculars from C upon the links. This, however, supposes that the links in their deviations are not sensibly removed from parallelism to the mean positions, and it would rarely be of any practical service.

former being to change the direction of motion of a link when that motion is limited in extent; whereas the latter is expressly formed to allow of unlimited rotation in the same direction. The bell-crank is analogous to the guide pulleys of wrapping bands (Art. 186), and accordingly these are sometimes employed in lieu of bell-cranks, to change the direction of motion of a link, by inserting at the place where the motion is diverted a piece of chain which passes over a guide pully.

206. If the given directions of motion intersect, as in fig. 116, we obtain four angles round the point of intersection, in two of which the directions of motion both approach the point, in another they both recede from it; and in the two remaining angles one motion approaches and the other recedes. The axis C may be placed in either of the two latter angles. If the directions of motion are parallel and opposite, the axis will lie between them, and if parallel and similar, the axis will lie beyond them, on one side or the other, but if also equal, then the axis is removed to an infinite distance, and the crank becomes practically impossible; but the change of motion may be effected by the next Article.

207. If the two directions of motion be not in one plane, let ad , cb , fig. 117, be these lines; find their common perpendicular dc ; draw ce parallel to ad , and in the plane bce construct the required crank, as in Art. 204, of which let B be the center, Bb , Be the arms respectively perpendicular to bc and ce .

Draw BA a common perpendicular to Bb and Be , and equal to dc . Draw Aa parallel and necessarily equal to



Be, then will *AB* be the axis, *Aa* and *Bb* the arms required to change the small motion in *ad* into the required motion in *cb*.

By a similar construction we can effect the change of a small motion in a given direction, into another *equal* motion in the same direction parallel to the first; which has been shewn to be impossible by the bell-crank in one plane, although the motions themselves are in one plane.

In the mechanism of organs, in which the transmission of such small motions is of frequent occurrence, the crank is termed a *back-fall* when its arms are in one horizontal straight line, and a *square* when they are at right angles. An armed axis, like fig. 117, is a *roller*, and the links are *stickers* when they act by compression, and *trackers* when by tension.