CHAPTER II.

OF THE MOTIONS OF PRIMARY MOVING PIECES IN MACHINES.

SECTION I.—General Explanations.

37. Frame; Moving Pieces, Primary and Secondary. (A. M., 427.)
—The frame of a machine is a structure which supports the moving pieces, and regulates the path or kind of motion of most of them directly. In considering the movements of machines mathematically, the frame is considered as fixed, and the motions of the moving pieces are referred to it. The frame itself may have (as in the case of a ship or of a locomotive engine) a motion relatively to the earth, and in that case the motions of the moving pieces relatively to the earth are the resultants of their motions relatively to the frame, and of the motion of the frame relatively to the earth; but in all problems of pure mechanism, and in many problems of the dynamics of machinery, the motion of the frame relatively to the earth does not require to be considered.

The moving pieces may be distinguished into primary and secondary; the former being those which are directly carried by the frame, and have their motion wholly guided by their connection with the frame; and the latter, those which are carried by other moving pieces, or which have their motion not wholly guided by their connection with the frame. For example, the crank-shaft and the piston-rod of a steam engine are primary moving pieces; the wheels of a locomotive are primary moving pieces; the connecting-rod of a steam engine is a secondary moving piece.

Connectors are those secondary moving pieces, such as links, belts, cords, and chains, which transmit motion from one moving piece to another, when that transmission is not effected by immediate contact.

38. Bearings (A. M., 428.) are the surfaces of contact of primary moving pieces with the frame, and of secondary moving pieces with the pieces which carry them. Bearings guide the motions of the pieces which they support, and their figures depend on the nature of those motions. The bearings of a piece which has a motion of translation in a straight line must have plane or cylindrical.*

* The word “cylindrical” is here used in the comprehensive sense, which denotes any surface generated by the motion of a straight line parallel to itself.
surfaces, exactly straight in the direction of motion. The bearings of rotating pieces must have surfaces accurately turned to figures of revolution, such as circular cylinders, spheres, cones, conoids, and flat discs. The bearing of a piece whose motion is helical, must be an exact screw. Those parts of moving pieces which touch the bearings should have surfaces accurately fitting those of the bearings. They may be distinguished into slides, for pieces which move in straight lines, gudgeons, journals, bushes, and pivots, for those which rotate, and screws for those which move helically.

The accurate formation and fitting of bearing surfaces is of primary importance to the correct and efficient working of machines.

39. The Motions of Primary Moving Pieces (A M., 429,) are Limited by the fact, that in order that different portions of a pair of bearing surfaces may accurately fit each other during their relative motion, those surfaces must be either straight, circular, or helical; from which it follows, that the motions in question can be of three kinds only, viz.:

I. Straight translation, or shifting, which is necessarily of limited extent, and which, if the motion of the machine is of indefinite duration, must be reciprocating; that is to say, must take place alternately in opposite directions: for example, the piston-rod of a steam engine.

II. Simple rotation, or turning about a fixed axis, which motion may be either continuous or reciprocating, being called in the latter case swinging, rocking, or oscillation. Continuous rotation is exemplified by the shaft of a steam engine; reciprocating rotation by various beams or levers.

III. Helical or screw-like motion, compounded of rotation about a fixed axis, and translation along that axis.

SECTION II.—Straight Motion of Primary Pieces.

40. Straight Translation is the motion of a primary piece sliding along a straight guiding surface. All the particles of the piece move through equal distances in a given time, along parallel straight lines; and the line joining any two particles remains unaltered in length and in direction.

41. Resolution and Composition of Motions.—The resultant of two or more component motions is the motion which results from putting them together. If the component motions are represented by straight lines, their resultant is found geometrically by joining together, end to end, a series of straight lines respectively equal and parallel to the given straight lines, and pointing in the same directions, and then drawing a straight line from the starting point to the further end of the series. For example:—
COMPONENT AND RESULTANT MOTIONS.

I. (See fig. 15.) To find the resultant of two component motions, A B and A C. Let the paper represent the plane of those motions. From B draw B D parallel and equal to A C, and pointing in the same direction; join A D; this will be the required resultant motion; or, in other words, complete the parallelogram, A B, D C; its diagonal, A D, will be the required resultant.

A motion may, if required, be resolved into components. The following are the cases most useful in mechanism:

II. (Fig. 15.) To resolve a given motion, A D, into components in two given directions in the same plane, A X and A Y.

Through D draw D C parallel to X A, cutting A Y in C; and D B parallel to Y A, cutting A X in B; A B and A C will be the required components.

III. (Fig. 16.) To resolve a given motion, A D, into one component parallel and another component perpendicular to a given direction.

Through A, parallel to the given direction, draw A X, upon which let fall the perpendicular D B; then A B will be the first of the required components, and A C parallel and equal to B D will be the second.

IV. Given, the traces of a plane (Article 15, page 6) and the projections of a straight line representing a motion (Article 11, page 4), to find the projections of two component motions, one perpendicular and the other parallel to the plane. By the rule of Article 31, page 13, draw the traces of a second plane parallel to the given plane, and traversing the point which represents one end of the given motion. Then by the rule of Article 33, page 13, find the projections of the perpendicular let fall on the second plane from the point representing the other end of the given motion. That perpendicular will be one of the required components; and the straight line from the first-mentioned point to the foot of the perpendicular will be the other. The lengths of the lines representing the component motions may be found, if required, by Article 19, page 7.

The component of the motion parallel to the given plane is obviously its projection on that plane. It is sometimes called the tangential component, and the component perpendicular to the given plane the normal component of the given motion.

V. Given, a resultant motion and one of two component motions, to find the other component motion. Combine the given resultant motion with a motion equal and opposite to the given component.
motion; the resultant of these two will be the required other component motion. For example, in fig. 15, let A D be the given resultant motion, and A B the given component; draw D C equal and parallel to A B, and pointing the opposite way; join A C; this will be the required other component: or otherwise, join B D and draw A C equal and parallel to it.

VI. (Fig. 17.) Given, the vertical projection, A B, and the horizontal projection, A' B', of a straight line representing a motion, to resolve that motion into three rectangular components parallel and perpendicular to the planes of projection. Let O X be the axis of projection (Article 9, page 4). Draw the straight lines A A', B B',

![Fig. 17.](image)

cutting the axis of projection (of course at right angles) in C and D. Then through any convenient point, O, in the axis of projection, draw the straight line Z O Y' at right angles to that axis; and take O Y' to represent a transverse horizontal axis, and O' Z to represent a vertical axis. (The point O is called the origin.) Then parallel to X O draw A' E' and B' E' to meet O Y', and A G and B H to meet O Z. The three components required will be represented by C D, E' F', and G H.

VII. Given (in fig. 17), the vertical projection, A B, and the horizontal projection, A' B', of a straight line representing a motion, to draw a third projection of the same straight line on a vertical transverse plane of projection perpendicular to the first two planes of projection. Construct fig. 17 as described in the preceding Rule. O Z and O Y' will be the traces of the third plane of projection. Produce X O towards Y"; then O Y" will represent the rabatment.
of O Y', and Z O Y" the rabatment of the vertical transverse plane upon the vertical longitudinal plane of projection. In O Y" take O' E" = O E, and O F" = O F'; draw E" A" and F" B" parallel to O Z, to meet A G and B H produced in A" and B" respectively; join A" B"; this will be the projection required.

According to the rule already stated in Article 19, page 7, the motion of which A B and A' B' are the projections is to be found by making K L = A' B', and joining L B, which line will represent the extent of the resultant motion.

The following are the relations between a resultant motion and its components as expressed by calculation. In fig. 15,—

\[
\sin C A B : \sin C A D : \sin D A B : : A D : A B : A C;
\]

also, \[A D^2 = A B^2 + A C^2 + 2 A B \cdot A C \cdot \cos C A B.\]

In fig. 16,

\[
A B = A D \cdot \cos B A D; \quad A C = A D \cdot \sin B A D;
\]

\[A D^2 = A B^2 + A C^2.\]

In fig. 17,

\[L B^2 = C D^2 + E' F'^2 + G H^2.\]

42. Relative Motion of Two Moving Pieces.—All motion is relative; that is to say, every conceivable motion consists in a change of the relative position of two or more points. In speaking of the motions of the moving pieces of machines, motions relatively to the frame are always to be understood, unless it is otherwise specified. It is often requisite, however, to express the motion of a point in a moving piece relatively to a point in the same or in another moving piece.

In the case considered in the present section, where the relative position of two points in the same moving piece remains unaltered, not only as to distance but as to direction, the relative motion of such a pair of points is nothing. The motion of one moving piece relatively to another is determined by the following principle:—Let P, Q, and R denote any three points; then the motion of R relatively to P is the resultant of the motion of R relatively to Q, combined with the motion of Q relatively to P; so that if the motions of Q relatively to P, and of R relatively to P are given, the motion of R relatively to Q is to be found according to Rule V. of the preceding Article, by compounding with the motion of R relatively to P a motion equal and opposite to that of Q relatively to P. For example, let P stand for the frame of a machine, and Q and R for two moving pieces which slide along straight guides;
and in a given interval of time let A B, in fig. 15, page 19, represent the motion of Q relatively to P, and A D the motion of R relatively to P; then A C, found by Rule V. of Article 41, will represent the motion of R relatively to Q.

In all cases whatsoever of relative motion of two bodies, the motion of one relatively to the other is exactly equal and contrary to that of the second relatively to the first. For example, let P and Q be two points; and when P is treated as fixed, let Q move through a given distance in a given direction relatively to P; then if Q is treated as fixed, P moves through the same distance in the contrary direction relatively to Q.

43. Comparative Motion (A. M., 385,) is the relation borne to each other by the simultaneous motions of two points, either in the same body or in different bodies, relatively to one and the same fixed point or body. It consists of two elements: the velocity-ratio, which is the proportion borne to each other by the distances moved through by the two points in the same interval of time; and the directional relation, which is the relation between the directions in which the two points are moving at the same instant.

In the case of two points in a primary piece whose motion is one of translation, the velocity-ratio is that of equality, and the directional relation that of identity; for all points in such a piece are moving with equal speed in parallel directions at the same instant.

When two points in two different pieces are compared, the comparison may give a different result. For example, let P, as before, stand for the frame of a machine, and Q and R for two moving pieces; and while Q performs relatively to P the motion represented by A B (fig. 15, page 19), let R perform relatively to P the motion represented by A D. Then the comparative motion of R and Q consists of the following elements:

\[
\text{the velocity-ratio, } \frac{A D}{A B};
\]

and the directional relation, represented by the angle B A D.

In most of the cases which occur in mechanism the motion of each point is limited to two directions—forward or backward—in a fixed path; so that the directional relation of two points may often be sufficiently expressed by prefixing the sign + or − to their velocity-ratio, according as their motions are similar or contrary; that is, the sign + denotes that those motions are both forward or both backward; and the sign − that one is forward and the other backward.

We may compare together the different components of the motion of one point, and the resultant motion. For example, in
figs. 15 and 16, page 19, the velocity-ratios of two component motions, as compared with their resultant, are expressed by

\[
\frac{AB}{AD} \text{ and } \frac{AC}{AD};
\]

and in fig. 17, page 20, the velocity-ratios of three rectangular component motions, as compared with their resultant, are expressed by

\[
\frac{CD}{LB'} \frac{EF}{LB'}, \text{ and } \frac{GH}{LB}.
\]

Strictly speaking, the principles of the geometry of machines, or of pure mechanism, are concerned with comparative motions only, and not with absolute velocities: or, in other words, those principles relate to the motions which different moving points perform in the course of the same interval of time, but not to the length of the interval of time in which such motions are performed. For example, in the case of a direct-acting steam engine, the principles of pure mechanism show that the piston makes one double stroke for each revolution of the crank; that the directional relation of the piston and crank-pin varies periodically, the piston moving to and fro, while the crank-pin moves continuously round in a circle; and that in particular positions of those pieces their velocity-ratio takes particular values; but the question of what interval of time is occupied by a revolution, or of how many revolutions are performed in a minute, belongs not to the geometry, but to the dynamics of machines. Further, in the case of a pair of spur wheels gearing into each other, the principles of pure mechanism show that in any given interval of time the numbers of revolutions performed by those wheels respectively are inversely as their numbers of teeth, and that the directions in which they turn are contrary; but those principles do not inform us how many revolutions either wheel makes in a minute.

44. Driving Point and Working Point.—The term driving point is used to denote that point, either in a whole machine or in a given moving piece of a machine, where the force is applied that causes the motion; and the term working point is used to denote the point where the useful work is done. These explanations contain references to the dynamics of machines; but it is to be understood that in the geometry of machines, or pure mechanism, it is the comparative motion only of the driving point and working point that is taken into consideration. It is to be observed, too, that the word “point” is here taken in an extended meaning; for the exertion of force or communication of motion at a mathematical point, of no sensible magnitude, is purely ideal; and when the word point is used with reference to the driving or the work of machines,
it is to be held to mean the *place* where the action that drives or that resists a machine is exerted, of what magnitude soever that place may be, whether a surface or a volume. Thus, the driving point in a steam engine comprehends the whole surface of the piston that is pressed upon by the steam which drives the engine; and the working point, where friction is overcome, comprehends the whole of the rubbing surface, and where a heavy body is lifted, the whole volume of that body. Nevertheless, for the sake of convenience in mathematical investigation, such places of the action of driving or resisting forces are often treated on the supposition that they may be represented by single points; for when such points are properly chosen, no error is incurred by making that supposition.

**SECTION III.—Rotation of Primary Pieces.**

45. **Rotation of a Primary Piece.** *(A. M., 370-372.)*—*Rotation or Turning* is the motion of a rigid body when lines in it change their directions; and it is the only kind of motion involving change of the relative positions of the particles of a body that is possible consistently with rigidity; that is to say, with the maintenance of the distance between every pair of particles in the body unchanged. An *axis of rotation* is a line in a rigid body whose direction is unchanged by the rotation; and a *fixed axis of rotation* is a line whose position, as well as its direction, is unchanged by the rotation. Every line in a rotating body which is parallel to the axis has its direction unchanged by the rotation. The rotation of a primary piece in a machine always takes place about an axis that is fixed relatively to the frame of the machine; that axis being the geometrical axis, or centre line, of a bearing surface (such as that of the journals or gudgeons of a shaft), whose form is that either of a circular cylinder or of some other surface of revolution. The *plane of rotation* is any plane perpendicular to the axis. Every such plane in a rotating body has its position unchanged by the rotation; and straight lines in such a plane—that is, straight lines perpendicular to the axis of rotation—change their directions more rapidly than any other straight lines in the same body.

46. **Speed of Rotation.** *(A. M., 373.)*—Although in the case of rotation, as well as in that of translation, the principles of pure mechanism are concerned with comparative velocities only, still it is desirable here to state, that the speed with which a rotating body turns is expressed in two different ways. For most practical purposes it is usually stated in turns and fractions of a turn in some convenient unit of time; such as a second, or (more commonly) a minute. For scientific purposes, and for some practical purposes also, it is expressed in *angular velocity*; which means, the angle swept through in a second by a line perpendicular to the axis of
rotation: that angle being stated in circular measure; which means the ratio of the length of the arc subtended by an angle to the radius of that arc. The following are examples of the values of angles in circular measure:

One degree, .................. 0.0174533 nearly;

A right angle, or quarter revolution, .................. 1.5708 nearly;

Two right angles, or half a revolution, ............... 3.1416 nearly = \frac{355}{113} very nearly.

Four right angles, or a revolution, .................. 6.2832 nearly = \frac{710}{113} very nearly.

Hence, to convert turns per second into angular velocity, multiply by \frac{113}{710} = 0.159155 nearly. The time of revolution in seconds is the reciprocal of the speed expressed in turns per second. The comparative speed or angular velocity-ratio of two rotating pieces is independent of the kind of unit in which their absolute speeds may be expressed; it is the reciprocal of the ratio of their times of revolution.

47. Rotation is Common to all Parts of the Turning Body. (A. M., 375.)—Since the angular motion of rotation consists in the change of direction of a line in a plane of rotation, and since that change of direction is the same how short soever the line may be, it is evident that the condition of rotation, like that of translation, is common to every particle, how small soever, of the turning rigid body, and that the angular velocity of turning of each particle, how small soever, is the same with that of the entire body. This is otherwise evident, by considering that each part into which a rigid body can be divided turns completely about in the same time with every other part, and with the entire body, and makes the same number of turns in a second, or a minute, or any other interval of time.

48. Right and Left-Handed Rotation. (A. M., 376.)—The direction of rotation round a given axis is distinguished in an arbitrary manner into right-handed and left-handed. One end of the axis is chosen as that from which an observer is supposed to look along the direction of the axis towards the rotating body. Then if the body seems to the observer to turn in the same direction in which the sun seems to revolve to an observer north of the tropics, the rotation is said to be right-handed; if in the contrary direction, left-handed; and it is usual to consider the angular velocity of right-handed rotation to be positive, and that of left-handed rota-
tion to be negative; but this is a matter of convenience. It is obvious that the same rotation which seems right-handed when looked at from one end of the axis, seems left-handed when looked at from the other end. In fig 18, the arrow $R$ represents right-handed rotation, and the arrow $L$ left-handed rotation. When a body oscillates about an axis its rotation is alternately right-handed and left-handed.

49. Translation of a Point in a Rotating Piece. (A. M., 377.)—Each point in a rotating piece (except those situated in the axis) has a motion of revolution—that is, translation in a circular path, round the axis of rotation; and the velocity of that translation is the product of the perpendicular distance of the point from the axis—that is, the radius of the circular path, into the angular velocity of rotation (Article 46, page 24). Thus, in fig. 19, let the

![Fig. 19.](image)

surface of the paper represent a plane of rotation; let $O$ be at once the trace and the projection of the axis of rotation on that plane, and $A$ the projection of a point in the rotating piece under consideration. Then the motion of that point (and of its projection $A$), takes place in a circle of the radius $OA$; and if $AA'$ be the arc described in a second, then

$$AA' = OA \times \text{angular velocity};$$

$$= OA \times \frac{710}{113} \times \text{number of turns per second};$$

also,

$$\text{angular velocity} = \frac{AA'}{OA}.$$
The velocity at a given instant of a point which moves in a curve, as distinguished from the arc traced in a second by that point, is represented by a straight line equal in length to that arc, and pointing in the direction in which the point is moving at the given instant; that is to say, being a tangent to the path of the point at that instant. Therefore, to represent by a straight line the velocity of the point now in question at the instant when its projection is at A, draw A a perpendicular to OA, and equal in length to AA' (= OA × angular velocity).

50. Motion of a Part of a Rotating Piece.—When what has just been explained is considered together with the statement in Article 47, it is easily seen, that if the centre of any part of a moving piece rotating about a fixed axis is situated in that axis, then the motion of that part is simply a rotation similar and equal to that of the whole piece; but if the centre of the part is situated at a distance, OA, from that axis, the motion of that part consists of a translation of its centre, with the velocity O A × the angular velocity, in a circle described about the fixed axis with the radius O A, combined with a rotation similar and equal to that of the whole piece, about a moving axis traversing A, and parallel to the fixed axis which traverses O. Consider, for example, that rotating piece in a steam engine which consists of the shaft, crank, and crank-pin, and which turns about the axis of the shaft, as a fixed axis of rotation, to which the axis of the crank-pin is parallel. Then the motion of the shaft consists simply in a rotation about its own axis; while the motion of the crank-pin consists in a translation of its centre, and of each point in its axis, in a circular path described about the axis of the shaft, combined with a rotation about its own moving axis similar and equal to that of the shaft. As an additional illustration, suppose one end of a cord to be held still, and the other to be attached to a hook which is fixed at the centre of a rotating wheel, and which therefore rotates along with and as part of the wheel. The cord will undergo one twist for each turn of the wheel. Now let the hook be removed from the centre, and fixed at any point in an arm of the wheel, or in its rim; the cord will still undergo one twist, neither more nor less, for each turn of the wheel; thus showing, as before, the effect of the rotation of the hook along with the wheel; and the only difference in the motion will be that the end of the cord attached to the hook will be carried round in a circle, at the same time that the whole cord is twisted. A secondary piece in a machine may be so contrived as to have translation in a circle or some other curved path without rotation; this will be considered in a later chapter.

51. Rules as to Lengths of Circular Arcs.—In connection with the motion of points in rotating pieces, and with various other
questions in mechanism, there is frequent occasion to measure the lengths of circular arcs, and to lay off circular arcs of given lengths. These processes may be performed by the help of calculation, and of the well-known approximate values of the ratio which the radius and the circumference of a circle bear to each other, viz:—

\[
\frac{\text{circumference}}{\text{radius}} = \frac{710}{113} \text{ nearly } = 6.283185 \text{ nearly};
\]

\[
\frac{\text{radius}}{\text{circumference}} = \frac{113}{710} \text{ nearly } = 0.159155 \text{ nearly};
\]

but it is often much more convenient in practice to proceed by drawing; and then the following rules are the most accurate yet known:—

I. (Fig. 22.) To draw a straight line approximately equal to a given circular arc, A B. Draw the straight chord B A; produce A to C, making A C = \( \frac{1}{2} \) B A; about C, with the radius C B = \( \frac{1}{2} \) B A, draw a circle; then draw the straight line A D, touching the given arc in A, and meeting the last-mentioned circle in D; A D will be the straight line required.

The error of this rule consists in the straight line being a little shorter than the arc; in fractions of the length of the arc, it is about \( \frac{1}{1600} \) for an arc equal in length to its own radius; and it varies as the fourth power of the angle subtended by the arc; so that it may be diminished to any required extent by subdividing the arc to be measured by means of bisections. For example, in drawing a straight line approximately equal to an arc subtending 60°, the error is about \( \frac{1}{1600} \) of the length of the arc; divide the arc into two arcs, each subtending 30°; draw a straight line approximately equal to one of these, and double it; the error will be reduced to one-sixteenth of its former amount; that is, to about \( \frac{1}{1600} \) of the length of the arc. The greatest angular extent of the arcs to which the rule is applied should be limited in each case according to the degree of precision required in the drawing.

II. (Fig. 23.) To draw a straight line approximately equal to a given circular arc, A B. (Another Method.) Let C be the centre of the arc. Bisect the arc A B in D, and the arc A D in E; draw the straight

* These rules are extracted from Papers read to the British Association in 1867, and published in the Philosophical Magazine for September and October of that year.
secant A E F, and the straight tangent A F, meeting each other in F; draw the straight line F B; then a straight line of the length A F + F B will be approximately equal in length to the arc A B.

The error of this rule, in fractions of the length of the arc, is just one-fourth of the error of Rule I., but in the contrary direction; and it varies as the fourth power of the angle subtended by the arc.

III. To lay off upon a given circle an arc approximately equal in length to a given straight line. In fig. 24, let A D be part of the circumference of the given circle, A one end of the required arc, and A B a straight line of the given length, drawn so as to touch the circle at the point A. In A B take A C = \( \frac{1}{4} \) A B, and about C, with the radius C B = \( \frac{3}{4} \) A B, draw a circular arc B D, meeting the given circle in D. A D will be the arc required.

The error of this rule, in fractions of the given length, is the same as that of Rule I., and follows the same law.

IV. (Fig. 24.) To draw a circular arc which shall be approximately equal in length to the straight line A B, shall with one of its ends touch that straight line at A, and shall subtend a given angle. In A B take A C = \( \frac{1}{4} \) A B; and about C, with the radius C B = \( \frac{3}{4} \) A B, draw a circle, B D. Draw the straight line A D, making the angle B A D = one-half of the given angle, and meeting the circle B D in D. Then D will be the other end of the required arc, which may be drawn by well-known rules.

The error of this rule, in fractions of the given length, is the same with that of Rules I. and III., and follows the same law.

V. To divide a circular arc, approximately, into any required number of equal parts. By Rule I. or II., draw a straight line approximately equal in length to the given arc; divide that straight line into the required number of equal parts, and then lay off upon the given arc, by Rule III., an arc approximately equal in length to one of the parts of the straight line.

Rule V. becomes unnecessary when the number of parts is 2, 4, 8, or any other power of 2; for then the required division can be performed exactly by plane geometry.

VI. To divide the whole circumference of a circle approximately into any required number of equal arcs. When the required number of equal arcs is any one of the following numbers, the division can be made exactly by plane geometry, and the present rule is not needed:—any power of 2; 3; \( 3 \times \) any power of 2; 5; \( 5 \times \) any power of 2; 15; \( 15 \times \) any power of 2.*

* It may be convenient here to state the methods of subdividing arcs and whole circles by plane geometry. (1) To bisect any circular arc. On the chord of the arc as a base, construct any convenient isosceles triangle,
cases proceed as follows:—Divide the circumference exactly, by
plane geometry, into such a number of equal arcs as may be re-
quired in order to give sufficient precision to the approximative part
of the process. Let the number of equal arcs in that preliminary
division be called \( n \). Divide one of them, by means of Rule V.,
into the required number of equal parts; \( n \) times one of those
parts will be one of the required equal arcs into which the whole
circumference is to be divided.

Rules I., III., and V., are applicable to arcs of other curves
besides the circle, provided the changes of curvature in such arcs
are small and gradual.

52. Relative Translation of a Pair of Points in a Rotating Piece.—
In fig. 19, page 26 (where O, as already explained, is at once the
projection and the trace of a fixed axis of rotation on a plane
perpendicular to it, and A the projection of a point in the rotating
piece), let B be the projection of another point in the rotating piece,
and A B the projection of the straight line connecting those two
points. The point B describes a circle of the radius O B about the
fixed axis; and the radii O A and O B sweep round with the
angular velocity common to all parts of the rotating piece, so that
by the time that A has moved to the position A', B has moved to
the position B', such that the angles A O A' and B O B' are equal.
In order to determine the motion of one of those moving points
(as A) relatively to the other (as B), it is to be considered that,
owing to the rigidity of the body, the length of A B is invariable,
and that the change of direction of that line (as projected on the
plane of rotation), consists in turning in a given time through an
angle equal to that through which the whole piece turns. In fig.
20, take B to represent at once the trace and the projection, on a
plane of rotation, of an axis parallel to the fixed axis, and traversing
the point B. Draw B A in fig. 20 parallel and equal to B A in
fig. 19; and B A' in fig. 20 parallel and equal to B A' in fig. 19.
Then A and A' in fig. 20 represent two successive positions of A
with the summit pointing away from the centre of the arc; a straight line
from the centre of the arc to that summit will bisect the arc. (2.) To mark
the sixth part of the circumference of a circle. Lay off
a chord equal to the radius. (3.) To mark the tenth
part of the circumference of a circle. In fig. 24 A,
draw the straight line A B = the radius of the circle;
and perpendicular to A B, draw B C = \( \frac{1}{2} \) A B. Join
A C, and from it cut off C D = C B. A D will be the
chord of one-tenth part of the circumference of the
circle. (4.) For the fifteenth part, take the difference
between one-sixth and one-tenth. It may be added,
that Gauss discovered a method of dividing the circumference of a circle by
gometry exactly, when the number of equal parts is any prime number that
is equal to \( 1 + \) a power of 2; such as \( 1 + 2^4 = 17; \ 1 + 2^8 = 257, \ &c.; \) but
the method is too laborious for use in designing mechanism.
relatively to the axis traversing B, at the beginning and end respectively of the interval of time in which the rotating piece turns through the angle \( \Delta O A' \) (fig. 19) = \( \Delta B A' \) (fig. 20). The translation of \( \Delta \) relatively to this new axis consists in revolution in a circle of the radius \( BA \), in the same direction with the rotation (that is, in the present example, right-handed); and the velocity of that relative translation is \( BA \times \) the angular velocity of rotation. Fig. 21 shows how, by a similar construction, the motion of \( B \) relatively to an axis traversing \( \Delta \) is represented. Take \( \Delta \) in fig. 21 to represent at once the trace and the projection, on a plane of rotation, of an axis parallel to the fixed axis, and traversing \( \Delta \). Draw \( AB \) and \( AB' \) in fig. 21 parallel and equal respectively to \( AB \) and \( AB' \) in fig. 19. Then \( B \) and \( B' \) in fig. 21 represent two successive positions of \( B \) relatively to the axis traversing \( \Delta \), at the beginning and end respectively of the interval of time in which the rotating piece turns through the angle \( \Delta O A' \) (fig. 19) = \( \Delta BA' \) (fig. 21); the translation of \( B \) relatively to this new axis consists in revolution in a circle of the radius \( AB \), in the same direction with the rotation (that is, in the present example, right-handed); and the velocity of that relative translation is \( AB \times \) the angular velocity, and is at each instant equal, parallel, and contrary to the velocity of translation of \( \Delta \) relatively to \( B \), agreeably to the general principle stated at the end of Article 42, page 21.

53. Comparative Motion of Points in a Rotating Piece.—In fig. 19, page 26, as before, let \( \Delta \) and \( B \) be the projections at a given instant, on a plane of rotation, of two points whose motions are to be compared. The directions of motion of those points at that instant are represented by the straight lines \( \Delta a, \ B b \), tangents to the circles in which the points revolve about the axis \( O \); and the directional relation of the points is expressed by the fact, that the angle between those directions of motion is equal to the angle \( \Delta OB \), between the perpendiculars let fall from the two points on the axis \( O \); or, in other words, the angle between the planes traversing that axis and the two points respectively; of which planes \( \Delta O \) and \( OB \) are the traces upon the plane of rotation; for the directions of motion, \( \Delta a, \ B b \), are respectively perpendicular to those two planes.

The velocity-ratio of the two points is equal to the ratio \( OB : OA \) borne to each other by the radii of their circular paths. In other words, if \( \Delta a = \Delta A' \) be taken, as before, to represent the velocity of \( \Delta \), and \( B b = B B' \) to represent the velocity of \( B \), then

\[
OA : OB :: \Delta a : \Delta b;
\]

and if the velocities of any number of points in a rotating piece are compared together, they are all proportional respectively to
the perpendicular distances of those points from the axis of rotation.

It is obvious that all points in a circular cylindrical surface described about the axis of rotation have equal velocities. The dotted circles in fig. 19, page 26, represent the traces of two such surfaces.

The relative motions of any two pairs of points in a rotating piece may be compared together. For example, let it be proposed to compare the motion of \( A \) relatively to \( B \) with the motion of \( B \) relatively to \( O \). Then, because the velocity of the motion of \( A \) relatively to \( B \) is proportional to \( B A \), and its direction perpendicular to the plane whose trace is \( B A \), while the velocity of the motion of \( B \) relatively to \( O \) is proportional to \( O B \), and its direction perpendicular to the plane of which \( O B \) is the trace, the directional relation is expressed by the angle made by those planes with each other, and the velocity-ratio by the ratio \( B A : O B \) borne to each other by the projections on the plane of rotation of the two lines of connection of the two pairs of points.

54. **Relative and Comparative Translation of a Pair of Rigidly Connected Points.**—The following proposition is applicable to all motions whatsoever of a pair of points so connected that the distance between them is invariable. It forms the basis of nearly the whole theory of combinations in mechanism, and many of its consequences will be explained in the ensuing chapters of this Part. At present it is introduced with a view to its application to pairs of points in a rotating piece.

![Fig. 25](image1)

![Fig. 26](image2)

**Theorem.**—If two points are so connected that their distance apart is invariable, the components of their velocities along the straight line which traverses them both must be equal; for if those component velocities are unequal, the distance between the points must necessarily change.

The straight line which traverses the points is called their Line of Connection.

For example, in fig. 25, let \( A \) and \( B \) represent two points in the plane of the paper, whose distance apart, \( A B \), is invariable. At a given instant let the velocities of those points be represented by straight lines, which may be in the same plane, or in different planes,
according to circumstances; and let $A\,a$ and $B\,b$ be the projections of those lines. From $a$ and $b$ let fall $a\,c$ and $b\,c'$ perpendicular to the line of connection, $A\,B$; these will be the traces of two planes perpendicular to the line of connection, and traversing respectively the points of which $a$ and $b$ are the projections; the parts $A\,c$ and $B\,c'$, cut off by those planes from the line of connection (produced where necessary), will be the components along that line of the velocities of $A$ and $B$ respectively; and those components must necessarily be equal—that is, $B\,c' = A\,c$. The component velocities transverse to the line of connection are represented by the lines whose projections are $c\,a$ and $c'\,b$, and may bear to each other any proportion whatsoever.

The same principle is illustrated in fig. 19, page 26. In that figure $A\,a$ and $B\,b$ represent the velocities of two points, $A$ and $B$, whose line of connection is $A\,B$, and is of invariable length; $a\,c$ and $b\,c'$ are perpendiculars let fall from $a$ and $b$ upon $A\,B$, produced where necessary; and $A\,c$ and $B\,c'$ represent the component velocities of $A$ and $B$ along the line of connection, which are equal to each other.

**Rule.**—*Given (in fig. 25), a pair of rigidly connected points, $A$ and $B$, and the directions of the projections $A\,a$ and $B\,b$ upon a plane traversing $A\,B$, of their velocities at a given instant, to find the ratio of those projections or component velocities to each other.* In fig. 26, draw $O\,c$ of any convenient length parallel to $A\,B$, and $a\,c\,b$ perpendicular to it; through $O$ draw $O\,a$ in fig. 26 parallel to $A\,a$ in fig. 25, and $O\,b$ in fig. 26 parallel to $B\,b$ in fig. 25; then the required ratio is

$$\frac{B\,b}{A\,a} = \frac{O\,b}{O\,a}.$$

55. **Components of Velocity of a Point in a Rotating Piece**—Periodical Motion. ($A.\ M.,$ 380.)—The component parallel to an axis of rotation, of the velocity of a point in a rotating body relatively to that axis, is nothing. That velocity may be resolved into rectangular components parallel to the plane of rotation. Thus let $O$ in fig. 27 represent the projection and trace of the axis of rotation of a body whose plane of rotation is that of the figure; and let $A$ be the projection of a point in the body, the radius of whose circular path is $O\,A$. The velocity of that point being $= O\,A \times$ angular velocity, let it be represented by the straight line $A\,V$ perpendicular to $O\,A$. Let $B\,A$ be any direction in the plane of rotation parallel to which it is desired to find the component of the velocity of $A$. From $V$
let fall $VU$ perpendicular to $BA$; then $AU$ represents the component in question. Sometimes the more convenient way of finding that component is the following:

From $O$ let fall $OB$ perpendicular to $BA$. Then $A$ and $B$ represent a pair of rigidly connected points; therefore, according to Article 24, the component velocities of $A$ and $B$ along $AB$ are equal. But $BA$, being perpendicular to $OB$, is the direction of the whole velocity of $B$; therefore the component, along a given straight line in the plane of rotation, of the velocity of any point whose projection is in that line, is equal to the whole velocity of the point where a perpendicular from the axis meets that line.

The whole velocity of $B$ is $= OB \times$ the angular velocity; and the velocity-ratio of $B$ to $A$, or, in other words, the ratio of the component velocity of $A$ along $BA$ to the whole velocity of $A$, is $OB : OA$.

The velocity of a point such as $A$ in a rotating piece may be resolved into components, oblique (see fig. 19) or rectangular (see fig. 27) as the case may be, by regarding the velocity of $A$ relatively to $O$ as the resultant of the velocity of $A$ relatively to $B$, and of that of $B$ relatively to $O$. The directions of that resultant velocity and its two components are respectively perpendicular to $OA$, $BA$, and $OB$, and their ratios to each other are equal to those of the lengths of the same three lines. This is a particular case of a more general proposition, viz.,—that the velocities of three points relatively to each other are proportional to the three sides of a triangle which make with each other the same angles that the directions of those three relative velocities do ($A. M., 355$).

In fig. 28, let $O$ be the trace of the axis on a plane of rotation, and $A$ a point in the rotating piece, revolving in the circle $OA$, so as to assume successively a series of positions such as 1, 2, 3, 4, 5, 6, 7, 8; and in each position of $A$, let the component velocity $AU$, parallel to a fixed plane whose trace is the diameter $8O4$, be compared with the whole velocity of revolution, $AV$. 
ROTATING PIECES—COMPARATIVE MOTION.

Let 2 O 6 be a diameter perpendicular to 8 O 4; and through A draw A Y parallel to 4 O 8 and A X parallel to 2 O 4. Then, according to the principles already explained, the value of the velocity ratio in question is,

\[ \frac{A \ U}{A \ V} = \frac{X \ A}{O \ A} = \frac{O \ Y}{O \ A}; \]

and it is evident that this ratio is equal to nothing when A is at the points 4 and 8, and to unity when A is at the points 2 and 6. Further, if the velocity of revolution be considered as always positive, and if the component velocity A U be considered as positive when from left to right, and negative when from right to left, the ratio \( \frac{A \ U}{A \ V} \) is,

- in the quadrant 812, positive and increasing;
- in the quadrant 234, positive and diminishing;
- in the quadrant 456, negative and increasing;
- in the quadrant 678, negative and diminishing.

It thus undergoes a series of periodical variations. All this is expressed in symbols by the formula,

\[ \frac{A \ U}{A \ V} = \sin \theta; \]

where \( \theta \) denotes the angle that the radius O A makes at any instant with the radius O 8.

Other and more complex ways of resolving the motions of points in rotating bodies into components will be considered in the next chapter.

56. **Comparative Motion of Two Rotating Pieces, and of Points in them.**—In comparing together the rotations of two rotating pieces without reference to the translations of points in them, their comparative speed is expressed (as already stated in Article 46) by the angular-velocity ratio, or ratio of the numbers of turns in a given time; which is also the reciprocal of the ratio of the periodic times of revolution. When the axes are parallel, or nearly parallel, the directional relation may be expressed simply by prefixing + or − to the velocity-ratio, according as the directions of rotation are similar or contrary; but there are cases to be considered further on, where the relative angular positions of the axes have to be considered with precision.

When the translations of two points in two different rotating pieces are compared, the directional relation is determined by the fact, that each point moves in a direction normal to a plane traversing itself and the axis about which it revolves; and that the velocity of each point is proportional to its perpendicular distance from that axis, and the speed of rotation about that axis,
jointly. Hence, let \( a \) and \( a' \) denote the angular velocities of two rotating pieces, or a pair of numbers proportional to those angular velocities; \( r \) and \( r' \), the perpendicular distances of a pair of points in those two pieces from their respective axes, or a pair of numbers proportional to those distances; and \( v \) and \( v' \), the respective velocities of those two points, or a pair of numbers proportional to those velocities; then the velocity-ratio of the points is,

\[
\frac{v'}{v} = \frac{a'r'}{ar}.
\]

In order that a pair of points in a pair of rotating pieces may have equal velocities—that is, in order that \( \frac{v'}{v} \) may be \( = 1 \), we must make the radii inversely proportional to the angular velocities—that is, \( a'r' = ar \), or \( \frac{r'}{r} = \frac{a}{a'} \).

**Section IV.—Screw-like Motion of Primary Pieces.**

57. **Helical or Screw-like Motion** ([A. M.], 382,) is compounded of rotation about a fixed axis, and of translation along that axis: the *advance* (as the translation in a given time is called) bearing a constant proportion to the rotation in the same time; in other words, the moving piece advances along the axis of rotation through an uniform length during each turn.

The subject of the resolution of screw-like motion into components in other and more complex ways will be considered in the next chapter.

58. **General Figure of a Screw—Pitch**, ([A. M.], 471.)—In order that a primary moving piece may have screw-like motion, its figure ought to be that of a true screw; and it ought to turn in a bearing of the same figure, fitting it accurately. The figure of a screw may be described in general terms as consisting of a *projection* of uniform cross-section called the *thread*, winding in successive coils round a circular cylinder. The best form of section for the thread of a screw that is to be used as a primary moving piece for producing helical motion only, and not as a fastening, nor in "screw gearing," is rectangular. The forms suited for other purposes will be considered later. There are two sorts of screws, convex, or *external*, and concave, or *internal*; in the former the thread winds round the outside of a cylindrical spindle; in the latter it winds round the inside of a hollow cylinder. When the word "screw" is used without qualification, an external screw is usually meant; an internal screw is called a "nut." When a primary moving piece is an external screw, its bearing is an internal screw; when the primary moving piece is an internal screw, the bearing is an external screw. The *truth* or accuracy of
figure of a screw depends mainly on the perfect uniformity of the pitch; that is to say, the distance, measured parallel to the axis, from any point in one coil of the thread to the corresponding point in the next coil. For example, the pitch of the screw R in fig. 29, so long as it is measured parallel to the axis, may be measured either from D to F, from E to G, from F to H, or from G to K, or between any pair of corresponding points in two successive coils; and it ought to be exactly the same wheresoever it is measured. The pitch is also the uniform distance through which the screw advances at each turn.

59. **Right-Handed and Left-Handed Screws.**—A screw is said to be right-handed or left-handed according as right-handed or left-handed rotation is required in order to make it advance; and this is a permanent distinction, and not dependent on the position of the spectator, as the distinction between right-handed and left-handed rotation is (Article 48, page 25). For example, in fig. 29, L is a left-handed screw, and R a right-handed screw.

Most screws used in the arts are right-handed; left-handed screws are made for special purposes only.

60. **Comparative Motion of a Point in a Screw.**—The principles of the present Article apply not only to any point in the thread or in the spindle of a screw, but to any point in a body that is rigidly attached to the screw, so as to move along with the screw as one piece; such as a wheel or a lever fixed to and turning with the screw. In fig. 29, let A B be the axis of the screw, and C a point rigidly attached to it at the perpendicular distance C A from the axis. Then, while the screw makes one turn the motion of the point C is the resultant of two components at right angles to each other: an advance, along with the whole screw, in a direction parallel to the axis, through a distance equal to the pitch of the screw; and a revolution, round a circle described about the axis with the radius A C, and having, therefore, the circumference \(6.2832\ A\ C\). In most questions of comparative motion connected with screws, the quantity of most importance is the velocity-ratio of those two components of the motion of a given point, and it is expressed as follows:

\[
\frac{\text{velocity of revolution}}{\text{velocity of advance}} = \frac{\text{circumference}}{\text{pitch}} = \frac{6.2832\ A\ C}{D\ F}.
\]
61. Path of a Point in a Screw—Linear Screw or Helix.—A point in, or rigidly attached to, a screw, traces a path which may be called a screw-shaped line or linear screw. By mathematicians it is called a helix. A helix winds round in successive similar coils upon a cylindrical surface described about the axis of rotation with a radius equal to the perpendicular distance of the tracing point from the axis. The distance, measured parallel to the axis, between any two successive coils is everywhere the same, and is identical with the pitch of the screw; and the angle of inclination of the linear screw to the axis is everywhere the same.

Points in, or rigidly attached to, a screw, at equal distances from the axis, trace by their motion equal and similar linear screws on one and the same cylindrical surface. Points at unequal distances from the axis trace different linear screws, inclined to the axis at different angles, and situated on cylindrical surfaces of unequal radii; but the pitch of all those linear screws is the same. All the edges, whether projecting or re-entering, of a screw-thread are linear screws.

A linear screw may be traced on a cylindrical surface by any mechanical contrivance which ensures that, while the cylinder rotates, the tracing point shall advance along a line parallel to the axis at a rate bearing a constant proportion to the rate of rotation. This will be further considered in that part of this treatise which relates to the construction of machinery.

A linear screw is the shortest line on the surface of a cylinder between two points that are neither in one plane traversing the axis nor in one plane perpendicular to the axis; and a cord or a flexible wire stretched on a cylindrical surface between two such points tends to assume of itself the figure of a linear screw.

62. Projection of a Linear Screw.—The most useful projection of a linear screw is that upon a plane traversing the axis, and is drawn as follows:—In fig. 30, let A B represent the axis of the screw. Draw D A C perpendicular to A B, making A C = A D = the radius of the cylindrical surface in which the helix is to be situated. Draw D I and C F parallel to A B; those two lines will be the traces of the cylindrical surface. About A, with the radius A C, draw the semicircle C K D; this represents the trace of one-half of the cylindrical surface on a plane perpendicular to its axis, "rabatted" upon the plane of projection. Divide the semicircle into any convenient number of equal arcs (Article 51, page 27); the greater the number of those divisions, the greater will be the accuracy of the projection. In fig. 29 the semicircle is divided into six equal arcs only; in practice a greater number will in general be required.

On C F, or any other line parallel to the axis, lay off C E = the intended pitch of the screw, and divide it into twice as many
equal parts as there are equal divisions in the semicircle C K D.

Through the points of division of the semicircle draw straight lines parallel to A B; and through the points of division of the pitch draw straight lines perpendicular to A B; the points of intersection of successive pairs of those two sets of lines will be points in the required projection of the linear screw, C G L E.
which can then be drawn through those points. This is the projection of one coil; and as many successive coils as may be required may be projected by simply repeating the same curve. In fig. 30 the projection, E H M F, of a second coil is shown; and it has been constructed by laying off an uniform distance parallel to the axis and equal to the pitch from each projected point of the first coil; for example, G H, L M, E F.

The half coils on the nearer side of the cylinder, viz., G L E and H M F, are drawn in thicker lines than the half coils, C G, E H, on the farther side of the cylinder. The screw is righthanded. Had it been left-handed, C G and E H would have been on the nearer side, and G E and H F on the farther side of the cylinder.

63. Development of a Linear Screw.—The development of any figure drawn on a curved surface consists in supposing the surface to be a flexible sheet, and drawing the appearance which the figure would present if that sheet were spread out flat on a plane. Some curved surfaces only are capable of being developed, such as a cylinder, and a cone; others, such as a sphere, are not. To draw the development of the cylindrical surface in fig. 30, as bounded by the two circles whose projections are C D and F I, draw C e perpendicular to A B, and equal in length to the circumference of the cylinder (see Article 51, page 27), and complete the rectangular parallelogram C e F; this will be the required development of the cylindrical surface.

To draw the development of one coil, C G E, of the linear screw, take e e = the pitch C E; draw the straight line C e; this will be the required development. To draw the development of the second coil, E H F, take e f = the pitch, and draw the straight line E f; and so on for any required number of coils.

The uniform angle of inclination of the linear screw to the axis is E C e = F E f.

One method of drawing a screw on a cylindrical surface is first to draw its development on a sheet of some flexible substance, and then to roll that sheet round a cylinder of the proper radius.

The several points in the development marked with small letters are the respective developments of the points marked with the corresponding capital letters in the projection.

To draw the development of the series of lines parallel to the axis which pass through the points of division of the circumference, divide C e into twice as many equal parts as the semicircle C D is divided into, and draw straight lines parallel to C F through the points of division, such as d i, q w, &c.

The length of one coil of the screw is

\[ C e = \sqrt{(\text{circumference}^2 + \text{pitch}^2)}. \]
64. The **Radius of Curvature** of a linear screw is found by the following construction:—In fig. 31, let \( AC \) be the radius of the cylindrical surface in which the screw is situated. Draw \( AY \), making the angle \( CAY \) equal to the angle which the screw makes with a plane perpendicular to its axis. Draw \( CY \) perpendicular to \( AC \), cutting \( AY \) in \( Y \), and \( YZ \) perpendicular to \( AY \), cutting \( AC \) produced in \( Z \). \( AZ \) will be the required radius of curvature. Its length may also be found by calculation, as follows:

\[
AZ = AC \left( 1 + \frac{\text{pitch}^2}{\text{circumference}^2} \right).
\]

65. **Normal Pitch.** (*A. M.*, 472.)—By the **normal pitch** of a linear screw is to be understood the distance from one coil to the next, measured, not parallel to the axis, but along the shortest line on the cylindrical surface between the two coils; that is to say, along another helix or linear screw which cuts all the coils of the original linear screw at right angles. The normal pitch is to be determined from the development of the screw, as follows:—In fig. 30, from \( c \) let fall \( ca \) perpendicular to \( Ce \); \( ca \) will be the normal pitch. The straight line \( ca \) is part of the development of the **normal helix**, as it may be called. When produced, it cuts \( Ef \), the development of the next coil, in \( o \), and \( no = ca \) is also the normal pitch. By finding the intersections, such as \( p \), of the development of the normal helix with the series of straight lines parallel to the axis, any number of points, such as \( P \), in the projection of the normal helix, \( CNO \), may be found if required. The normal pitch may be calculated by the following formula:

\[
\frac{ca}{ce} = \frac{\text{circumference} \times \text{pitch}}{\text{length of one coil}}.
\]

The **pitch of the normal helix**, if required, may be found by producing \( cp \) in fig. 29 till it cuts \( CF \) produced, and measuring the distance of the point of intersection from \( C \); and then its radius of curvature may be found by a construction like that in fig. 30; but in general it is more convenient to find these quantities by calculation, as follows:

\[
\text{pitch of normal helix} = \frac{\text{circumference}^2}{\text{pitch of original helix}}
\]

\[
\text{radius of curvature of normal helix} = AC \left( 1 + \frac{\text{circumference}^2}{\text{pitch of original helix}^2} \right).
\]

The sum of the reciprocals of the radii of curvature of the original helix and the normal helix is equal to the reciprocal of the radius of the cylinder; that is, to \( \frac{1}{AC} \).
The pitch of a screw as measured parallel to the axis may be called the *axial pitch*, in order to distinguish it from the normal pitch; but when the word "pitch" is used without qualification, axial pitch is always to be understood.

The several linear screws which exist in the figure of an actual solid screw, or which are described by points in it or rigidly attached to it, have all the same axial pitch; but they have not the same normal pitch except when they are situated on the same cylindrical surface.

66. **Divided Pitch.**—A screw sometimes has more than one thread, in which case the distance between any coil of any one thread and the next coil of the same thread is divided by the other threads into as many parts as the total number of threads. In that case the distance from a point in one thread to the corresponding point in the next thread may be called the *divided pitch*, to distinguish it from the distance between two successive coils of the same thread, or pitch proper, which may, when required, be designated as the *total pitch*. The advance of the screw at each revolution depends on the total pitch only, in the manner already explained, and is wholly independent of the number of threads and of the divided pitch; so that division of the pitch does not affect the motion of a screw as a primary piece. Its use in combinations of pieces will be afterwards explained.

Division of the pitch of a linear screw is illustrated in fig. 30, where two additional linear screw threads, marked by dotted lines, are shown dividing the pitch of the original screw into three equal parts. To avoid confusion, one only of the additional screw-lines is lettered, viz., that marked $Q R S T U V W$ in the projection, and $q r, R s t u, U v w$, in the development. The other is unlettered. The *divided axial pitch* is $C R = \frac{1}{3} C E$, and the divided normal pitch $c x = \frac{1}{3} c u$.

The several linear screw threads in a case of this kind are all parallel and similar to each other; and in the development they are represented by parallel straight lines. They divide the circumference into as many equal parts as there are threads; and the length of one of those parts may be called the *circular* or *circumferential pitch*. In fig. 30, the circular pitch is represented by the arc $C Q$, and by its development $c q$. 