CHAPTER XII.

FRICTION IN MECHANISMS AND MACHINES.

§ 71.—FRICTION.

WHEN two surfaces are pressed together it is found that one cannot be moved along and relatively to the other, without the exertion of some definite effort. The resistance, to balance which this effort has to be exerted, is called the **friction** between the surfaces. It can be measured as a force acting from one surface to the other in the direction

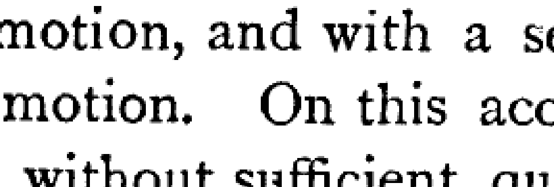

FIG. 326.

of their relative motion, and with a sense such as to offer resistance to that motion. On this account it is sometimes loosely spoken of, without sufficient qualification, as a force which tends always to oppose and never to produce motion.[1] Let *b* and *c* (Fig. 326) be two bodies touching one another, and let *b* slide upon *c* (supposed fixed) under the action of the

[1] A proposition somewhat fiercely attacked by Reuleaux, *Kinematics of Machinery*, p. 594.

force MN. This force has a component MP pressing the surfaces together and causing friction, and a component PN in the direction of motion. Let the external resistance to the motion of b, independently of friction, be QN. Then, disregarding friction, the body b will be receiving an acceleration of $\frac{PN-QN}{m}$ foot-seconds per second, its mass being supposed to be m. If now Q_1Q be the frictional resistance produced by the pressure component MP, the body will be receiving an acceleration only of $\frac{PN-(QN+Q_1Q)}{m}$ foot-seconds per second, and the difference between these two values is the acceleration caused by the frictional resistance. If $Q_1N = PN$ the body b will be moving with a uniform velocity, just as it would do if the friction were absent and QN were equal to PN. We do not say in such a case that the resistance QN does not produce motion; we treat it, on the contrary, as a force in every respect similar to the effort PN, but differing from it in sense. There seems no really sufficient reason for treating frictional resistances in any different manner.

A frictional resistance has always a sense opposite to that of the relative motion of the bodies between which it acts. So far, therefore, as the motion of these bodies relatively to each other is concerned, the acceleration produced by it is always negative. But it is often utilised in order to produce positive acceleration of one of the bodies relatively to a third. Fig. 327 illustrates this, where c is not itself a fixed body, but one capable of sliding upon a fixed body d. Suppose that b were fixed to c by bolts whose united resistance to fracture was RN. Then if the effort could exceed this value the bolts would shear and b would move upon c. But so long as the effort PN is less than RN, the two bodies could not move relatively to each other, and PN would be balanced by a stress in the bolts (that is, by a *portion* of RN), exactly equal to itself, as QN. The stress in the bolts is what may be called a **derived force**, which has a maximum value RN, but whose actual value is any magnitude less than this which is necessary to balance the external force opposed to it, here PN. This precisely represents the conditions of the case if we substitute frictional resistance between the surfaces for the shearing resistance of the supposed bolts. The friction is a derived force depending here upon the pressure MP, and upon the state of the surfaces. It has some maximum value (which we may suppose to be RN) entirely independent of PN. If PN exceed that value the bodies

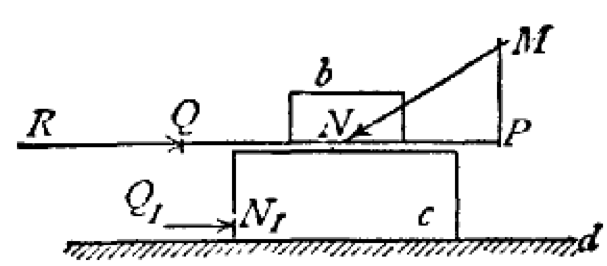

FIG. 327.

will move relatively to each other as in Fig. 326. If PN falls short of that value it will be balanced by a frictional resistance exactly equal to itself, the rest of the possible frictional resistance being non-existent in the same sense as the balance of possible stress in the bolts. Hence if RN be in this case the maximum possible friction, and PN the only driving effort, b will remain stationary relatively to c, under the equal and opposite forces PN and QN. But if c can move relatively to d under some sufficiently small resistance (frictional or other), Q_1N_1, it will be set in motion, receiving the acceleration $\frac{PN - Q_1N_1}{m}$ foot-seconds per second, just as before. In this case it seems legitimate to say that friction is the cause of *positive* acceleration and not negative, for it is the friction between *b* and *c* which transfers the driving effort *PN* from *b* to *c*, and which in that sense is the cause of the positive acceleration of *c* relatively to *d*.

Both the cases described occur continually in machinery. Wherever two surfaces have to be rubbed together (as in every pin joint or other pair of elements throughout the whole machine), frictional resistances cause negative acceleration, work is expended in overcoming them, they diminish the efficiency of the machine, and it is the object of the engineer to reduce them to the furthest possible extent. In all belt and rope gearing, however (and in a few other cases), the frictional resistance between two bodies (the belt and the pulley) is utilised as the sole means of giving motion to one of them (relatively to a third) and transmitting work to it. It is essential for this purpose that the possible friction between the two bodies (as *b* and *c* in Fig. 327) should be as *great* as possible, and its magnitude only affects the efficiency of the machine if it is too small, so as to permit the relative motion of the bodies which it is intended to prevent.

Experiments made upon the friction of bodies caused to slide upon one another without any, or with little, lubrication, at very moderate velocities, and with small intensities of pressure,[1] have established the facts that under these conditions the friction is independent of the area of contact and intensity of pressure, and is practically independent of the velocity of rubbing, being for any given pair of surfaces proportional simply to the total normal pressure. Under such conditions, therefore, the frictional resistance can be found at once for any known value of the pressure *P* by multiplying it by some co-efficient μ dependent essentially on the

[1] By *intensity* of pressure is meant, as formerly, pressure per unit of area.

nature of the surfaces, so that the value of the friction is written

$$F=\mu P.$$

The multiplier μ is called the **co-efficient of friction**, and is assumed to be fairly constant for given materials with such surfaces as are commonly used.

Engineers, however, have seldom to do with unlubricated rubbing surfaces, and they have to deal with surfaces moving often with very high velocities, and under very great and frequently varying pressures. Under these conditions the "laws" of friction, as they have just been stated, not only do not hold exactly true, but fail even to represent approximately the more complex phenomena with which they have to deal. At many speeds and loads which are of daily occurrence in machinery, velocity and intensity of pressure have an enormous effect on the friction, and not only these, but the temperature of the surfaces and the nature of the lubricant. The nature of the rubbing contact also, whether continuously in one sense or continually reversed, whether the surfaces be flat as in a guide, or cylindrical as in a bearing, whether contact exist throughout a surface or only along a line, very greatly affects the friction. The actual material of which the surfaces consist forms only one out of an immense number of conditions which determine friction under a given load. In fact, although all the rubbing surfaces in a machine were made of the same material, and had as nearly as possible the same smoothness, the co-efficient of friction, that is the quantity by which the total pressure on each surface would have to be multiplied to find the friction, instead of being practically constant, might be ten times[1] as great for some pairs of surfaces as for others. In

[1] Often enormously more than ten times. The particular number ten is not intended to have any special significance.

each particular pair of surfaces, with its own special area, velocity, form, amount of lubrication, and so on, the frictional resistance bears a different proportion to the load, and can be estimated from it only by the use of a different co-efficient. Under these circumstances it is perhaps misleading to retain the much-used phrase, "co-efficient of friction," for this inconstant multiplier. The co-efficient of friction has the certain definite meaning which has already been explained, and which limits its use to solid friction under certain simple conditions. It is so thoroughly associated with the idea that friction is proportional to load, that it seems unadvisable to call by its name a mere multiplier which may even itself vary inversely as the load. We shall, therefore, speak rather, in the following sections, of the **friction-factor** for a given pair of surfaces, meaning by this expression simply the ratio, dependent on all the varying conditions already mentioned, of the frictional resistance of those surfaces to the pressure causing it. We may, therefore, write $\frac{F}{P}=f$, the friction-factor, so that we still have

$$F=fP$$

but with the condition that f is a quantity whose value has to be separately considered for each set of conditions.

In every mechanical combination, from a pair of elements to a machine, some effort is at each instant expended in balancing friction, some work therefore is done, as the machine moves, merely in overcoming frictional resistance. If we call the remaining effort or work, as the case may be, the *nett* or *useful* effort or work of the combination, the ratio $\frac{\text{useful effort}}{\text{total effort}}$ or $\frac{\text{useful work}}{\text{total work}}$ is called the **efficiency** of the apparatus. Where the ratio is between amounts of *effort* only, we have the efficiency simply at the instant and for the position at which the effort has been measured. Where the ratio is between quantities of *work*, it gives us the *average* efficiency during the period in which that work has been done.[1] The reciprocal of the efficiency was called by Rankine the **counter-efficiency**, a name of great value. The counter-efficiency expresses, of course, the ratio in which it is necessary that the whole effort exerted or work done should exceed the nett value of the effort or work required.

The older results as to friction rest mainly on the experiments of Morin (dating as far back as 1831), which were most carefully conducted, and the results of which, within the limits and under the conditions to which they are fairly applicable, there is no reason whatever to doubt.[2] But they were made under conditions which, however well they may represent those of ideal solid friction (as they were intended to do), do not at all represent those of ordinary machinery. In spite of the numerous experiments of Thurston and others, of the brake trials of Westinghouse and Galton, and of the recent experiments of a Research Committee of the Institution of Mechanical Engineers made by Mr. Tower, we have still not

[1] The idea of the efficiency of a machine, now so familiar, we appear to owe to Moseley (*Phil. Trans.* 1841, and *Mech. Principles of Engineering*, 1843). He called it the *modulus* of the machine, and worked out its value in an immense number of different cases, including that of toothed gearing, spur and bevel.

[2] For Morin's experiments on sliding friction, see the *Mémoires de l'Institute* for 1833, which contain two series. The friction was measured between blocks of various materials and flat rails of the same or of different materials. The blocks were heavily loaded, and the motion (having been first started by special apparatus) was kept up by the pull of a descending weight. In some cases the velocity was uniform, in most accelerated. In no case could the experiment last more than a few seconds. The velocity and the amount of the pull were registered automatically. The distance through which sliding occurred was from ten to twelve feet. Morin's experiments on *Frottement des axes de rotation* were made in 1834.

nearly sufficient information to enable us to give probable values for the friction-factor under many of the most important cases occurring in practice.

Let us take first friction in journals or pin joints generally, assuming that the one surface moves continuously over the other, and does not reciprocate. What we call the "pressure on the bearing" does not here represent the actual pressures between the surfaces, but rather the total value of the components of those pressures in a certain direction. As it is only this nominal total pressure that forms part of our data in practice, it is sufficient to accept it as a starting point, without troubling ourselves here as to the real distribution of pressure. Engineers often speak of the pressure *per square inch* upon a bearing, by which they invariably mean the total pressure divided by the area of the bearing upon a plane normal to it, that is, by the product of the length of bearing and diameter of shaft. This nominal "pressure per square inch" is, therefore, an entirely conventional unit. Tower's experiments, which were made upon a steel journal four inches diameter and six inches long, give the remarkable result that for a given speed the total friction remains nearly constant for all ordinary loads [1] not too great nor too small for the particular lubricant used, so long as the lubrication was kept "perfect." The friction-factor, therefore, varied *inversely as the load.* It varied at the same time directly (very nearly) as the square root of the velocity. The formula

$$f = 20\, c \frac{\sqrt{v}}{P}$$

expresses very closely the results of these experiments (so long as the lubrication was kept perfect) for a temperature

[1] These experiments did not go below 100 pounds per square inch nominal. *Proc. Inst. M. Engs.* 1883 and 1884.

of 90° Fahr. v is the peripheral velocity of the bearing in feet per minute, and P the nominal pressure per square inch upon it. c is a co-efficient depending on the lubricant, and has a value of ·0014 for sperm oil (up to 300 pounds per square inch pressure), ·0015 for rape oil, and ·0018 for mineral oil (up to about 450 pounds per square inch), ·0019 for olive oil (up to 520 pounds per square inch), and about ·003 for mineral grease (between 150 and 625 pounds per square inch). The value of $20\frac{\sqrt{v}}{P}$ is unity (nearly) at a speed of 250 feet per minute and a pressure of about 310 pounds per square inch, so that c is itself the friction-factor, or co-efficient of friction, for these conditions. It will be noticed how much smaller it is than the value usually taken.

When the lubrication was not made "perfect" by the use of an oil bath, but the oil supplied, as regularly as was possible, by a syphon lubricator, the friction-factor was about four times as great as that given. It followed the same law as to variation inversely as the pressure, but its variation with velocity was much less than before, and was irregular. When the lubrication was reduced to a minimum ("so that the oiliness was only just perceptible to the touch"), it was increasingly difficult to get uniform results, but those that were obtained approximated distinctly, as was to be expected, to the usually assumed conditions of solid friction. Between loads of 100 and 200 pounds per square inch the friction-factor diminished as the load increased, but much less rapidly, and from 200 to 300 pounds per square inch (at which pressure "seizure" occurred), the factor remained nearly constant, forming a real co-efficient of friction varying only from ·008 to ·010. The variation with the velocity was larger at the lowest pressures, but smaller and irregular afterwards.

Temperature was found to affect the friction very greatly. With a load of 100 pounds per square inch, for instance, the friction with lard oil was about double as much at 75° as at 120°, and about three times as much at 60° as at 120°. No doubt there is a best possible temperature for each lubricant at each load, namely, that temperature which keeps it as thin as possible without making it so liquid as to be squeezed out.

Mr. Tower seems to have shown beyond doubt that with perfectly lubricated journals the metal surfaces should be, and are, separated by a film of lubricant,[1] and this fact at once explains the immense discrepancies between the results just stated and those obtained in such experiments as Morin's.

Nominal pressures of 200 to 500 pounds per square inch are common in the bearings of machinery, but in certain cases, such for instance as the pin in a piston rod, head pressures of 800 to 1,200 pounds per square inch are constantly used without any ill effects. In these cases, however, the speed of rubbing is slow[2] (quite possibly slower than that to which the friction diminishes with the velocity), the motion is reciprocating, and above all the pressure is alternating in direction, first on one side of the pin, then on the other. We have no experiments on friction under these conditions, but know by experience that bearings working in this fashion can carry a very much greater load than those loaded in one direction and revolving continuously under the load. The crank pin of an engine

[1] *Proc. Inst. M. Engs.*, 1883, etc.

[2] At *excessively* slow speeds the experiments of Jenkin and Ewing have shown that friction increases as the velocity diminishes, until (probably) the friction of motion (with which only we concern ourselves here) merges continuously into the friction of rest. But these speeds are much slower than any we have to deal with.

forms an intermediate case, the velocity of rubbing may be about the same as for the shaft, but the pressure is alternately in opposite directions. In the crank shaft of an engine, and still more in ordinary shafting, the weight of fly wheel, pull of belts, etc., cause the general direction of pressure to remain comparatively unaltered. Correspondingly the pressure in such cases is made considerably less than in a crank pin, although the velocity of rubbing is about the same.

The friction in the ordinary pin joints of linkwork, where the lubrication is not so well attended to as in shaft bearings, must vary enormously. As long as the lubrication is uniform, even if it is very small, it ought to be possible to work them with a friction-factor of ·010 to ·015, remaining approximately constant at such loads as they can carry. With freer lubrication a value for the friction-factor of

$$f = 0{\cdot}015\,\frac{100}{p} = \frac{1{\cdot}5}{p}$$

may approximately represent what can be obtained.

As to the friction of such lubricated flat surfaces as guide blocks, there appear to be no modern experiments. Those of Morin, already mentioned, give for sliding metal blocks, with "lubricant constantly renewed," a true co-efficient of friction of 0·05,[1] varying neither with the velocity nor with the pressure. This was at various velocities and at pressures averaging 28 pounds per square inch, and sometimes as much as 110 pounds. Ordinary steam-engine guide-blocks have a velocity (in alternate directions) varying in each stroke from 0 to 500 or more feet per minute, and under these circumstances it is found that they work best when the maximum pressure upon them is kept under 40 or

[1] This figure is given by Morin himself in connection with his 1834 experiments, as representing his very best possible results, but it is considerably lower than those results themselves, in which, no doubt, the lubrication was very imperfect.

50 pounds per square inch, which points to a friction-factor very much higher than for bearings.

On the friction of pivots there is very little experimental evidence. The case is complicated by the fact that the velocity of rubbing varies from zero to a maximum over the surface, from the centre outwards, and that the distribution of pressure varies also (see § 75) in a way which we do not know. Here, as in cross-head pins, it is found practically possible to allow often a larger pressure than in ordinary bearings, and an average of 700 and 800 pounds per square inch, and in some cases double as much, can be carried without injury.

As to the friction-factor for the rubbing in higher pairs, such as wheel teeth, there is also exceedingly little experimental evidence. The brake experiments of Westinghouse and Galton [1] show that at equal velocities and pressures the friction-factor for the wheels skidding on the rails was only about one-third as great as for the wheels rubbing on the brakes. The comparison is between rubbing with *line* contact (as with higher pairs) and rubbing with *surface* contact. If these results are applicable to the lubricated, or semi-lubricated, higher pairings which occur in machinery, their friction-factor must be much lower than it would otherwise be assumed to be, and this is probably the case.[2] If the smallness of the surface causes a reduction of the friction where there is no lubrication, it is probable that it may cause still more where even imperfect lubrication exists.

In the case of higher pairing, as with toothed wheels, cams, &c., we have seen (§ 18) that the relative motion of the surfaces is not pure sliding, but is equivalent to a combination of rolling and sliding, the particular lines which are in

[1] *Proc. Inst. M. Engs.* 1878 and 1879.

[2] This conclusion is strongly corroborated by recent experiments by Mr. John Goodman in the author's laboratory at University College.