THE KINEMATICS OF MACHINERY.

Most of the models used to illustrate this and the following lecture belong to the Kinematic Collection of the Gewerbe-Akademie in Berlin, and have been designed by Professor Reuleaux, who is the Director of the Academy and a Professor in it. The rest were sent to the Loan Collection by Messrs. Hoff and Voigt of Berlin, and Messrs. Bock and Handrick of Dresden. In essentials there is no difference between the Berlin and the Dresden models. Both have been designed specially for use in instruction in the Kinematics of machinery.

I must first try to explain briefly, but exactly, what I mean by the phrase "Kinematics of machinery." Professor Reuleaux, whose models are before us, defines a machine as "a combination of resistant bodies so arranged that by their means the mechanical forces of nature
can be compelled to do work accompanied by certain determinate motions." The complete course of machine instruction followed in some of the Continental technical schools covers something like the following ground:

First, there is the perfectly general study of machinery, technologically and teleologically. Then there comes what we may call the study of prime movers, which in terms of our definition would be the study of the arrangements by means of which the natural forces can be best compelled to do the required work. Then comes the study of what may be called "direct actors," or the direct-acting parts of machinery; in the terms of our definition, the arrangement of the parts of a machine in such a way as best to obtain the required result. Next comes what we call machine design; the giving to the bodies forming the machine the requisite quality of resistance. Machine design is based principally on a study of the strength of materials.

One clause of the definition still re-
mains untouched. The machine, we said, does work accompanied by certain determinate motions. Corresponding to this we have in machine instruction the study of those arrangements in the machine by which the mutual motions of its parts, considered as changes of position only, are determined. The limitation here must be remembered; motion is considered only as a change of position, not taking into account either force or velocity. This is what Professor Willis long ago called the "science of pure mechanism," what Rankine has called the "geometry of machinery," what Reuleaux calls "kinematics," and what I mean now by the "kinematics of machinery."

The results of many years' work of Reuleaux in connection with this subject are embodied in his book *Die Theoretische Kinematik*, which I recently had the pleasure of translating, and I shall endeavor to give you an outline of his treatment of the subject. It cannot be more than an outline, as you will readily
understand. The subject is a very large one, and I have had to choose between taking up many branches of it and merely mentioning each, and confining myself to a few points, and going more into detail about them. I have chosen the latter plan, believing that the former would be of little benefit to anybody. It will be easy for those who are sufficiently interested in the matter to follow it up, and to study those parts which I omit, by the aid of the book I have just mentioned. My lecture to-day will be principally theoretical, and to-morrow I shall go more into practical applications. So far as possible, as I have Professor Reuleaux's models before me, I shall endeavor to follow his own order in treating the subject.

I presume you are acquainted, to a certain extent, with the ordinary method of studying "pure mechanism;" the method originated by Monge (1806), developed in Willis' well-known *Principles of Mechanism* (1841), and made popular, to a great extent, by Prof. Goodeve's
capital little text book and others. Each mechanism is studied for and by itself, in general, by the aid of simple algebraic or trigonometric methods, and is spoken of in reference to a certain "conversion" of motion which occurs in it. Thus, we have the conversion of circular into reciprocating motion, the conversion of reciprocating into circular, &c., and simple formulæ express certain relations between the motions of two or more moving points. In this way we know something important about a great number of mechanisms, and arrive at many results which are both useful and interesting. Some things are still left wanting, however; and these things may be summed up in this way:

(1.) We notice at once that we have taken the mechanism as a whole. We do not analyze it in any way whatever, and therefore,

(2) We have scarcely any knowledge of its relations with other mechanisms, or (what is quite as important) of the various forms which one and the same
mechanism may take. We shall see presently how extraordinarily various these forms are. We have never a general case with special cases derived from it; each case is treated by itself as a special one. Then

(3) The mechanism is studied in general from a point of view which gives us only the conditions of the motion of two points in it, or two portions of it, and is then left. The kinematic conditions of the mechanism as a whole remain absolutely untouched.

In such a mechanism as that of an ordinary steam engine, for instance, we study the relative motions of the guide block and the crank, or, I ought, perhaps, to say of the axes of the cross head and of the crank pin. We thus know the motions of two points in the rod which connects those axes, the "connecting rod," but we leave the motions of its other points untouched. It may, of course, be said that these others are of much less practical importance. This is true to some extent, although their practi-
cal importance is greater than might be supposed at first. But in any case these motions must certainly be studied if we are to obtain a complete knowledge of the mechanism to which they belong. Any method of study, therefore, which covers all the kinematic conditions of the mechanism, instead of the mechanical conditions of two or three points only, possesses in that respect very great advantages.

The treatment of mechanisms which I shall sketch to you, is intended to remedy some of the defects which I have enumerated. Those of you who have studied modern geometry, side by side with the old methods, will recognize that these defects are somewhat analogous to those of Euclidean geometry. The attempt to remedy them proceeds in lines similar to those of modern geometry, and will eventually, I believe, when more fully worked out, take the same position in its own subject.

Let us, then, look first at the analysis of mechanisms. This is none the less
important a matter that its results are so very simple in many cases. A clear understanding of those elementary matters is of great assistance in clearing up difficulties which occur in the more advanced parts of the subject.

In a machine or a mechanism of any kind the motion of every piece must be absolutely determinate at every instant. It will be remembered that we are at present considering motion as change of position only, not in reference to velocity. The motion of change of position may be determined by the direction and magnitude of all the external forces which act on the body; the motion is then said to be free, but it is obviously impossible to arrange such a condition of things in a machine. The motions may, however, be made absolutely determinate independently of the direction and magnitude of external forces; and in order that this may be the case, the moving bodies, or the moving and fixed bodies as the case may be, must be connected by suitable geometric forms. Motion, under
these circumstances, is called constrained motion.*

If I allow a prismatic block to slide down the surface of an inclined plane its motion will be free; it is determined by the combination of external forces which act upon the block. If the block be pressed on one side as it slides, it at once moves sideways, and can only be kept in a straight path if directly the pressure is exerted on the one side an equal and opposite force (or a force which has a resultant with the first in the direction of motion) be caused to act upon it on the other. If, on the other hand, the block be made to slide between accurately-fitting grooves (like a guide block in a machine), inclined at the same angle as the plane, and like it fixed, the block may be pressed sideways or in any other direction, but no alteration in its motion can take place; the motion is "constrained," it can occur

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*Essentially it does not differ from free motion; the difference really lies in the substitution of stresses or molecular forces, which are under our complete control, for external forces.
only in the one direction permitted by the guiding grooves. In the one case the external force has to be balanced by another external force; in the other the balancing force is molecular, i.e., is a stress and not an external force, and comes at once into play the instant the disturbing force is exerted. The geometric forms which are used in this way to constrain or render determinate the motions in machines are very various, and are chosen in reference to the particular motion required. If every point in a body be required to move in a circle about some fixed axis, a portion of the body is made in the form of a solid of revolution about that axis, and this is caused to "work in" another similar solid; the two forming the familiar pin and eye. If all points of a body be required to move in parallel straight lines we get, similarly for guiding forms, a pair of prisms of arbitrary cross section; a slot and block. If every point of a body be required to move in a helix of the same pitch we use a pair of screws
of that pitch, one solid and one open, for constraining the motion—a screw and nut.

The general condition common to these very simple forms is that, in each case, the path of every point in the moving body is absolutely determined at every instant, that is to say, the change of position of the moving body is absolutely determinate.

The geometric name for these mutually constraining bodies is envelopes, and each one is said to envelope the other. We shall call them (kinematic) elements, and the combination of two of them we shall call a pair of elements.

Those we have mentioned are special and very familiar and important cases of pairs of elements, which are of great simplicity. They have the common property of surface contact, the one enclosing the other, and are therefore called closed or lower pairs of elements. They are, moreover, the only closed pairs which exist. They are, further, the only pairs in which all points of the moving element have similar pairs.
Every point of an eye, for instance, moves in a circle about the same axis. If there were attached to it a body of any size or form whatever, all its points would move about the same axis. The "point paths" would all be concentric circles. Again, whatever the external size or shape of a nut, every point in it moves in a helix of the same pitch about the axis of the screw; the point paths, that is, would be similar.

The general condition of determinateness of motion can, however, be fulfilled by an immense number of other pairs of elements. The theory of these is too large a subject to be entered into just now, I must merely direct your attention to the existence of such combinations.

Fig. 1 represents one of the simplest that can be used. Here one of the elements is an equilateral triangle, ABC, the other is the "duangle" RPSQ. The latter moves within the former, touching it always in three points, or rather along three lines. Its motion is just as absolutely determinate as the
motion of a pin in an eye. It is free to move at any instant only about the point in which the three normals to the triangle at the points of contact intersect (as Q in the Fig.). The models before you show a few of the many forms taken by such pairs of elements. It is worth

![Diagram](image)

while noticing a few points in which the motions determined by them differ from the motions of the closed pairs. First, as we have already seen, the contact of the elements determining the motion was surface contact in the former
case, while here it takes place only along a finite number of lines. Then the motions of all points in the first case were similar; in these pairs the motions of the points are not similar, but entirely dissimilar, the motion of each point depending entirely upon its position. Fig. 2 shows a few of the point paths of the pair of elements shown in Fig. 1. The strikingly different curves obtained from one pair of elements, according to the choice of the describing point, is too obvious to need further notice.*

These pairs of elements are called higher pairs. They have only a few applications in practice, their interest being chiefly theoretical. From our present point of view their theoretic interest is considerable, because of their exact analogy with the lower pairs.

There is another difference between the two kinds of pairs which deserves notice, for reasons which will be better

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*The triangle UTQ and the three curves within it, which have M₃ for their center, are point paths. The curve triangle and the duangle shown in thicker lines will be explained further on.
understood afterwards. The pair of elements determine the relative motion of the two bodies connected by them. If one body be stationary on the floor or the earth, the moving body has the same motion relatively to the floor or earth that it has to the other element. If I move about both bodies in my hand, both have motion relatively to the earth, but the relative motion of the one to the other remains unchanged. It is of course only a case differing in degree from the former one, for in the former one both bodies had the motion of the earth itself, while one had the additional motion which I gave it. We may, however, not to be pedantic, speak of anything as “fixed,” or “stationary” which has the same motion as the earth.

Now, (in this sense) we may fix either element of a pair, and with the lower pairs the relative motion taking place remains the same whichever element be fixed. With the higher pairs, on the other hand, the relative motion is altered, and the point paths become en-
tirely different. The point paths of the duangle relatively to the triangle are, for instance, quite different from those of the triangle relatively to the duangle. This change of the fixed element is called the inversion of a pair.

The ultimate result of our analysis of mechanisms is then pairs of elements; we cannot go below this. The pairs we have noticed are of two kinds, each having their own definite characteristics. If, now, two or more elements of as many different pairs be joined together we get a combination which is called a (kinematic) link. It is obvious that the form of such a link is, kinematically, absolutely indifferent. The choice of its form and material belongs to machine design. It may be brick and mortar, cast iron, timber, as we shall see afterwards, but the fact that this is indifferent, kinematically, cannot be too distinctly kept in mind.

We can make combinations of links by pairing the elements which each contain to partner elements in other links, and
such combinations are called *kinematic chains*. Thus, if we denote similar elements by similar letters, \( aa, bb, cc, \&c. \), and the link connection by a line, we may indicate some of the chains obtainable from four pairs and four links, thus:

\[ a-bb-cc-dd-a \]

(we suppose the "chain" to return on itself and the two elements \( a \) to be paired, the whole forming a *closed chain*); or,

\[ a-cc-bb-dd-a \]

or

\[ a-dd-cc-bb-a \&c. \]

For the sake of illustration we give in

![Fig. 3](image-url)

Fig. 3 a sketch of a familiar chain containing four links, each connected to the adjacent link by a cylinder pair of ele-
ments. The axes of the four pairs of elements are parallel.

We have, then, in the kinematic chain, a combination so constructed that all its parts have determinate motions, motions absolutely fixed by the form of the elements carried by its links, and independent (considered as changes of position) of the application of external force. To convert the chain into a mechanism we have only to do what we have already done in connection with pairs of elements, fix one element—or, as each element is rigidly connected with a link, we may say preferably fix one link. Any link may be fixed, the chain, therefore, gives us as many mechanisms as it has links. In general these are different, in special cases only two or more of them are the same. We shall be able to enter into this part of our subject at some length in the next lecture; at present it will suffice to note two or three of the leading characteristics of chains and mechanisms which we can now easily recognize. These are,
(i.) That the motion of any link relative to either adjacent link is determined by the pair of elements connecting them.

(ii.) That the motion of any link relative to any other than its adjacent links depends on all the elements of the chain.

(iii.) That no link of a mechanism can be moved without moving all the other links except the fixed one, and

(iv.) That there can be only one fixed link in a mechanism.

The two last propositions require a few words of explanation. Suppose that in any combination of, say, four links, two can be moved without moving the other two, the combination is actually one of three links only, for clearly the two immovable links may be made into one, and are two only in name. This is very often the case in machinery, where special mechanisms are frequently used for the express purpose of connecting rigidly two or more links, and making them act as one, at certain intervals.
If, however, in the combination supposed, one link be fixed, while two can be moved and the fourth can either move or be stationary, the combination no longer comes under our definition of *constrainment*, for the motions are at a certain point indeterminate, at the point, namely, when it is possible for the fourth link either to move or to stand. Chains often occur in which this would be the case, were it not that mechanicians take means, either by adding other chains or in other ways, to constrain the motion which would otherwise be useless to them.

We have now obtained some idea of the way in which mechanisms are formed, of the elements of which they consist. Before applying the knowledge we have thus acquired I must direct your attention to some geometric propositions which will greatly facilitate the theoretic dealing with these mechanisms.

In order that I may not enter into too wide a subject, I shall confine myself here to the consideration only of "con-
plane" motions, or motions in which all points of the moving body move in the same plane or in parallel planes. The limitation is a large one, but the cases included under conplane motion cover the greater part of those which occur in practice. The method which I have to describe is equally applicable to general motion in space as to simple constrained conplane motions of which I shall speak.

Let me remind you that the motion of any figure moving in a plane is known if the motion of any two points (i. e. of a line) in it be known. The motion of any body having conplane motion is known if the motion of a plane section of it, parallel to the plane of motion, be known. Such a plane section of it is, of course, simply a plane figure moving in its own plane. The motion of any body having conplane motion (as in nine cases out of ten in machinery) can, therefore, be determined by the determination of the motion of two points. In speaking now, therefore, of the motion of a line for shortness' sake, it must be remembered
that we are really covering all cases of conplane motion of *solid bodies*.

In Fig. 4 PQ and $P_iQ_i$ are two positions of the same plane figure, or plane section of a body having conplane motion. If now we have two positions (in the same plane) of any plane figure, we know that the figure can always be moved from the one to the other by turning about some point in the plane. The position of the point O, about which the figure can be turned from the position PQ to the position $P_iQ_i$ can be found at once by the intersection of the normal bisectors to PP, and QQ. The motion of PQ in the plane is, of course, its motion relatively to the plane, and therefore relatively to any figure (as A B) in the plane. Such a point O as we have found here is called a *temporary center*, because the turning or motion takes place about it for some finite interval of time. It will be remembered that not only the two points and PQ of the figure, but every other point of it, must have a movement about this same point O at the
same time. Now suppose we have some further position of the same figure, as for example at the position marked $P_2Q_2$, we can find in the same way the center about which the figure must be turned to move from $P_1Q_1$ to $P_2Q_2$. We may indicate this point as $O_1$. Similarly taking other positions of this figure $P_3Q_3$ and so on, we can find other points, $O_2O_3$, &c. By joining the points $OO_1O_2O_3$, we obtain a polygon, and if the figure in its motion come back to its original position the polygon also comes back on itself, and passes again through the point $O$. Such a polygon, whether it be closed in this way or not, is called a central polygon; its corners are the temporary centers of the motion of the figure.

I have pointed out that all the points in the figure $PQ$ move round $O$ during the motion from $PQ$ to $P_1Q_1$. They move round $O$ necessarily through some particular angle, the angle $POP_1$, and every point moves through the same angle, which we may call $\varphi_1$. As the figure may have any form we choose, let
us suppose it so extended as to contain a line which is the same length as $OO_1$, and which makes with $OO_1$ the angle $\varphi_1$, that is to say, the angle through which the figure moves about $O$. Such a line is shown in Fig. 4 by $MM_1$.

We have, then, a line $MM_1$ forming a part of the figure $PQ$, equal in length to $OO_1$, the points $O$ and $M$ coinciding, and the angle $O_1MM_1$ being $= \varphi_1$. Then when the figure has completed its motion about $O$, $MM_1$ and $OO_1$ must coincide. Take further similarly $M_1M_2 = O_1O_2$ and so placed that when $M_1$ coincides with $O_1$, $O_2M_1M_2 = \varphi_2$, then when the figure takes its third position, completing the turning about $O_1$, $M_1M_2$ coincides with $O_1O_2$. Similarly we can obtain $M_2, M_3, \&c$. The figure thus found is another polygon, which we may call a second central polygon.

These polygons have important properties, the principal of which can be very easily recognized. The first polygon does not alter its position during the motion of the body; it is therefore fixed,
so that it may be considered as a part of any figure such as AB, which is fixed or stationary in the plane of motion. The second polygon moves with PQ and forms (by construction) part of the same figure with PQ. This second polygon then, by the consecutive turnings of its corners upon the corresponding corners of the first (and equal-sided) polygon, will give to PQ the required changes of position relatively to the fixed plane or to the figure AB lying in it.

If, therefore, we know the central polygons for the given motion, we know not only the changes of position of the points P and Q, but those of every other point connected with the moving figure, whatever form it may have. For at any one instant every point in the figure is moving about the same center. In studying the relative motions of the figures we may, therefore, quite leave out of sight their form if we only know the central polygons for the motion. These tell us, so far, all about the motion which is taking place.
We may go further, however. We have recognized the fact that the relative motion of two figures or bodies may take place equally whether one or the other of them be fixed, or both moving. In the case before us we have supposed AB fixed and PQ moving relatively to it. The second polygon then moves on the first, and expresses the relative motion taking place. If, however, we suppose PQ fixed and AB moving, then the polygons still express the relative motion; but the second is now fixed and the first rolls upon it. This follows directly from the constitution of the polygons. The properties of the polygons as expressing the relative motions of the bodies to which they belong are, therefore, reciprocal.

You will have noticed, no doubt, that the polygons do not express continuous motion. They define only a series of changes of position in their beginning and end, not telling us of the intermediate stages.

We may, however, take the consecu-
tive position of the figures as close to-
gerther as we like. The closer together they are taken the shorter become the sides of the polygons. If at last the distances $PP_1, P_1P_2, QQ_1, \&c.$, be taken, infinitely small, each corner of the polygon will be infinitely close to the next one. That is to say the two polygons will become curves, and of these curves “infinitely small parts of equal length continually fall together after infinitely small rotations about their end points.” In other words the two curves roll on one another during the continuous alterations in the relative position of the two figures. Instead of finding points now by the intersection of normal bisectors, they are found by intersection of normals to the paths of $P$ and $Q$ (Fig. 5). The turning about each point now occurs (not in general) for a finite period, but for an instant only. Each point is therefore called an instantaneous center. The curve containing all the instantaneous centers, or the locus of instantaneous centers, is called a centroid. Without
giving them any special name, several writers on Mechanics have made more or less use of these curves. Among these I

may mention Dwelshauvers-Dery, Schell and Pröll. Reuleaux has, however, given them a name \((Polbahnen)\), and has made some special use of them, more,
I think, than has been made by former writers.

While the polygons only represent a series of isolated positions of a body, the centroids, rolling on each other, represent the whole motion continuously. Like the central polygons their properties are reciprocal. If then the centroids of two figures be known, their relative motions for a series of changes of position, each infinitely small, are also known, i.e., their motions are completely determined.

If AB and its centroid be fixed, and the centroid of PQ rolled upon it (Fig. 5), we have now the means of determining the path of motion of every point in the Fig. PQ relative to AB, whatever may be the form of PQ. It is sometimes of great convenience to be able to find the motions of all points in a body in such a very simple way. Reciprocally we can determine the point paths of AB relatively to PQ, which, in general, differ entirely from those of PQ relatively to AB.
If both figures be moving, as frequently happens in practice, both centroids are also in motion; their motion relative to each other, however, remains unaltered. They still roll on one another, and their point of contact is still the instantaneous center of the motion of each relatively to the other. Each figure moves, relatively to the other, about this point, which being common to the two centroids, is common to the two figures. They might, therefore, for the instant, be connected at that point by a cylindric pair of elements. There are many problems of which the solution is greatly simplified by the recollection of this fact. The point in each figure which coincides with the instantaneous center, has, therefore, no motion relatively to the other figure. We have already seen this in the special case where the one figure is stationary, for then the point in which the moving centroid touches the fixed one is, by hypothesis, also stationary for the instant; in other words, it has no motion relatively to the fixed centroid. We now
see the general condition of which this is a special case.

Fig. 6 shows the centroids for the higher pair of elements of Fig. 1. The curve triangle UTQ is the centroid of the triangle ABC, and the shaded duan-

gle PVQW is the centroid of the duangle RPSQ.* As the duangle moves in the triangle (the elements *sliding* upon each other), its centroid *rolls* within the centroid of the triangle. Both centroids are

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*These centroids are shown on a larger scale, apart from the elements to which they belong, in Fig. 2.*
in this case formed of arcs of circles, and all the point paths (being determined by the rolling of one circular arc upon another) are combinations of trochooidal arcs.

The centroids of kinematic chains are generally of greater complexity than those of the pairs of elements just mentioned, but in some cases are quite as simple. In Fig. 7, for example, is shown a mechanism familiar to engineers, in which a crank $a$ drives a reciprocating bar $c$ by means of a block $b$ working in a slot. The centroids defining the relative motions of the links $a$ and $c$ are the two circles shown in full lines, one double the diameter of the other. These two circles both move as the mechanism works (supposing the link $d$ to be fixed), but always so that they roll continuously one on the other. If instead of fixing $d$ the crank $a$ were made the fixed link, the same centroids would still express the relative motions of $a$ and $c$. The smaller circle, the centroid of $a$, would be stationary along with the link to which it
belongs, and the other would roll on it, the instantaneous center for the motion of the link $c$ being always at their points of contact. This mechanism ($a$ being fixed) is used in Oldham's coupling, in elliptic chucks, &c. Knowing these cen-
troids we know all about the motions of the two corresponding links in the mechanism, not only about the motions of some particular points in these links.

The centroids of kinematic chains can in general be very easily determined. Once found they make us independent to a great extent of trigonometric or algebraic formulæ, and enable us to determine all we wish to know by purely geometric graphic constructions. For technical purposes, at least, this is frequently an immense advantage. There are very few cases in which it is not more convenient for the engineer to employ a construction than a formula, if both give him the same result.

Before looking at the centroids of other mechanisms, it is necessary to examine one particular case which often occurs. Suppose that the lines PP, and QQ, in Fig. 4, or the tangents to the curves at P and Q in Fig. 5, had been parallel. It is obvious that the normal bisectors in the one case and the normals to the curve in the other then become
also parallel, or, as it is for some reasons more convenient to express it, would meet at an infinite distance. The temporary center in the one case and the instantaneous center in the other are at infinity. A centroid may, therefore, contain one or more points at an infinite distance, may have, that is, one or more infinite branches. This constantly occurs in mechanism, and in some cases every point in the centroid is at an infinite distance. This is, however, a special case; its treatment does not offer any practical difficulty, but I cannot do more than mention its existence here.

The centroids of the connecting rod and frame of the ordinary steam engine driving mechanism (the links $b$ and $d$ of Fig. 8) may serve as an illustration of
this. When the crank $a$ is at right angles to $d$, the normals to the paths of the two points 2 and 3 are parallel. The instantaneous center of $b$ relatively to $d$ is, therefore, at an infinite distance. Each centroid has, therefore, a pair of infinite branches.

We may look, in conclusion, at one other case which possesses some special interest on account of the form taken by the centroids. It is shown in Fig. 9. The chain contains four links and four parallel cylinder pairs. The alternate links are equal, and the two longer links are crossed so that the chain forms an "anti-parallelogram" in every position, the angle at 2 being always equal to that at 4, and the angle at 1 to that at 3. If the link $d$ be fixed, the links $a$ and $c$ become two cranks which revolve in opposite directions with a varying velocity ratio. The centroids of $b$ and $d$ are a pair of hyperbolae having their foci at 2 3 and 1 4 respectively. The one rolls upon the other as $b$ moves, the instantaneous center in the position shown be-
ing at the point of contact O, which is the point of intersection of 1 2 and 3 4. The centroids of the two shorter links are the two ellipses which are shown in dotted lines. They are confocal with the hyperbolæ, and their point of contact is always at the intersection of 1 4 and 2 3. Their form shows at once that the rotation of the axes 1 and 4 is precisely the same as that which would be communicated by a pair of elliptic spur wheels having the centroids for their pitch ellipses.

In this mechanism, as in some of the others illustrated, the centroids of two adjacent links, as a and d or b and c, are simply a pair of coincident points which roll upon each other. They form thus a limiting case of centroids, but every theorem which applies to the more extended centroidal curves applies also to these points, as can easily be seen on examination.