3 KINEMATIC MODELS

3-1 THE MEANING OF MODELS

The term *model* evokes many interpretations if considered in detail. In the large sense of the word, however, there is the implication that a model—whatever its form—establishes communication and furnishes an understanding transcending the written word. Model has nothing to do with size: it has to do only with perceptive form.

Models of varied kinds have been used for millennia. Depending on their purpose, models may express ideas of design, confirm or negate an assumption or hypothesis, guide analytical reasoning away from a false path, obtain fundamental data, and so on.

In the study of models we shall be principally concerned with hardware types suitable for kinematics. These represent a language of a very special kind, quite distinct from the conventional symbolic notations that have been devised for convenience and manipulative procedures. Models are a visual language. The figures of a geometer are models: with them the relations of points, lines, angles, and planes are kept track of. But ordinary geometry is static, one diagram being sufficient. Kinematics is geometry in motion: there is a new diagram for each instant. Displacement, the first aspect of motion, demands a movable geometric figure capable of displaying (1) the changing relations of angles and line lengths, (2) the position changes of points, and (3) the curves traced by the points. It is one thing to show a mechanism in a given position, and quite another to show several phases, separating each phase in the welter of lines. In fact, the time spent in making a model may very well be less than that required to pursue graphically a motion through the several phases judged to be critical or of interest. Good and bad transmission angles will discover themselves, unexpected change points may appear, and the like. A model certainly furnishes the quickest way to the display of coupler-point curves.

While Lagrange's epoch-making "Méchanique analytique" was further distinguished by a total lack of figures, most persons require diagrams and sketches to define the geometrical relations according to which their work proceeds. Kinematic models able to demonstrate the displacement relations (among other things) consequent to any position are the kinematic equivalents of the geometer's diagrams and the mechanicist's figures.

With a spatial mechanism the drafting problem may verge on the unspeakable because of the maze of construction lines and the need for auxiliary views. Spatial models, more difficult to construct than their planar counterparts, are nevertheless well worth the trouble, for situations defying the imagination are put on display.

Kinematic models fall into three categories. The first group is composed of models that the instructor demonstrates to the class: all members of the class may or may not follow the motions as they come and go. The models of the second group are those which are available for personal manipulation and study ("fussing"). The third and most rewarding category embraces all models, no matter how crude or rude, made by the interested individual. This group, and we denote it as the "do-it-yourself" variety, is the most valuable of the lot. The virtue and merit come from the fact that the man who builds his own models—who nurses them along and makes them work—has lived with the mechanism and learned its moods. Because of this intimacy the model builder learns much more than the model viewer or model fusser.

Fortunately, a great number of planar models of even considerable complication can be made on a desk top from materials and office equipment at hand: no machine-shop facilities are required. Cardboard strips, thumbtacks, scissors, knife, adhesive, etc., are the matériel and tools. Spatial models, having links subjected to bending and torsion and connections such as ball-and-socket joints and cylindrical pairs, require metal[1] and somewhat more in the way of a "shop" facility, but no high-grade shop with elaborate tools. The model maker, in using whatever

[1] It is possible to fold torsionally and flexurally stiff links from cardboard, but metal strips, if available, are more satisfactory.

material is convenient, will have to supply ingenuity, patience, and time. However, after gaining a little experience, the time expended on a model will be reduced and a simple model produced with little effort. Another way of putting this is to say that not only can standard situations be investigated—those presented in a book or paper, say—but different proportions of the links (lengths) may also be looked at. All this is much more satisfying and instructive than being bound to a prepared model of invariant proportions (e.g., the first and second categories mentioned earlier) or trying to follow link motions on a drawing. This is not to say that drawing-board work can be dispensed with; on the contrary, the drawings become much more meaningful. Models augment drawings; they do not supplant them.

It is of these do-it-yourself models that we shall speak after having considered something of the general background of kinematic model work.

3-2 HISTORICAL SURVEY

Geometric models of various sorts have an ancient history. For example, Archimedes (third century B.C.) weighed models of parabolic segments before developing the theorem that the parabolic area is two-thirds of the area of the circumscribing parallelogram. Archimedes is also known to have made a planetarium of some kind in which the motions of the sun, moon, and planets were displayed. We note that this early kinematic model was concerned with showing the relative motions of *points*. More elaborate devices of the same nature, but operating by wheelwork and stemming from the eighteenth century, are known as orreries.

Nonkinematic models of the past and present are legion. Architects and builders of medieval times made models of their cathedral structures to plan the erection of the stonework. For centuries ships were built by the scaling up of dimensions taken from hand-carved hulls. In the elaborate "Admiralty models," details such as interior framing, decks, and most other structural features were included. For the seventeenth century one may read that it had become the "custom of the times" to develop large-scale undertakings from the miniature. To do more than mention the models used by all phases of the aircraft industry would be to belabor the point.

An early, perhaps first, discussion of kinematic models stems from Robert Willis (1800–1875), the kinematician of Cambridge. Willis was the successor to the Rev. William Farish, a remarkable man. Farish (1759–1837) occupied the chair of chemistry before being elected to the office of Jacksonian professor of natural and experimental philosophy in the year 1813. We read in Willis (1851) that Farish "soon after commenced a Course of Lectures on Arts and Manufactures, which he repeated yearly until his death in 1837. The plan of this Course included the exhibition of almost all the more important machines which were then in use in the manufactures of Britain. This led him to conceive the possibility of devising a system of mechanical apparatus consisting of the separate parts of which machines are made, so adapted to each other, that they might admit to being put together at pleasure in the form of any machine that might be required."[1] Willis goes on to say that by 1837, when he himself became Jacksonian professor, "it appeared . . . that his [Farish's] idea of a Protean mechanism was capable of being carried out in a different and more complete manner, so as to be of greater practical utility, and of a more extensive application to philosophical apparatus in general."

Few details are known about the Farish "system," but the pieces were chiefly made of metal, some of the steel shafting being of $\frac{3}{4}$-in. octagon section. Willis said:

> The entire plan . . . had the merit of great simplicity and ingenuity, but the machines were apt to appear somewhat embarrassed and complicated by the variety of clamps and junctions of the framework. It is to be regretted that the ingenious inventor of this mechanism did not draw up a detailed account of his system. The general appearance of his models may be gathered from the sketch of an optical grinding engine appended to his Paper on Isometrical Perspective. . . .

This is Fig. 3-1. In this paper Farish proposed the now widely used isometric perspective of 30°, derived from sighting down the diagonal of a cube. He coined the name *isometric*, since the three perpendicular dimensions of the cube were equal.

Building on the experience of his predecessor, and taking advantage "of the facilities which the improved state of machine-making afforded," Willis devised a rather complete system, or kit, of parts,[2] giving all dimensions, naming the pieces, and so on. He recommended that each setup, once achieved and in operating order, be sketched in isometric projection for the record.

The various parts of the Willis kit were also substantial, since

[1] These lectures became part of the curriculum designed to liberalize education in the classics!

[2] It seems that these kit parts have not been preserved. Such models as are ascribed to Willis—and a few still exist at Cambridge—are made of materials and components quite different from those described so glowingly in 1851.