CHAPTER III.

HYDRAULICS.

PRELIMINARY REMARKS.

THE science which treats upon the mechanical properties and effects of water and other fluids, has most commonly been divided into two branches, HYDROSTATICS and HYDRAULICS. *Hydrostatics* treats of the weight, pressure, and equilibrium of fluids, when in a state of rest. *Hydraulics* treats of water in motion, and the means of raising, conducting, and using it for moving machinery, or for other purposes. These two divisions are so intimately connected with each other, that the latter could not be at all understood without an acquaintance with the former; and it is not necessary, in a work like the present, to treat of them separately. Considered abstractedly, the same laws obtain in the pressure and motion of water, as those which belong to solid bodies; and in the last chapter, on Mechanics, this similarity has led to some notice of the effects produced by water, which, strictly speaking, would belong to the present. In doing this, utility has been preferred to a strict adherence to system.

In treating of the elementary principles of Hydraulics, it is necessary to proceed upon theoretical principles; but let it always be recollected that from various causes resulting from the constitution of fluids, and particularly from that essential property in them, the perfect mobility of their particles among each other, the phenomena actually exhibited in nature, or in the processes of art, in which the motion of water is concerned, deviate so very considerably from the deductions of theory, that the latter must be considered as a very imperfect guide to the practical mill-wright and engineer. It is not to be inferred from this circumstance, that such theoretical investigations are false and useless; they are still approximations, which serve as guides to a certain extent. Their defectiveness arises from our inability to form an estimate of the many disturbing causes which influence the motion of fluids; whilst in the mechanics of solids we have, in many cases, no other correction to make in our theoretical deductions, than to allow for the effect of friction.

"The only really useful method of treating a branch of knowledge so circumstanced, is to accompany a very concise account of such general principles as are least inapplicable to practice, by proportionately copious details of the most accurate experiments which have been instituted, with a view to ascertain the actual circumstances of the various phenomena." (*Lardner*'s *Hydrostatics*. Such has been the course pursued, to a considerable extent, by the author of this work, and in pursuing this subject, under the present head of Hydraulics, we shall consider only such parts of the science as immediately relate to our purpose: namely, such as may lead to the better understanding of the principles and powers of water, acting on mill-wheels, and conveying water to them.

ARTICLE 45.

OF SPOUTING FLUIDS.

Spouting fluids observe the following laws:—

1. Their velocities and powers, under equal pressures, or equal perpendicular heights, and equal apertures, are equal in all cases.*

2. Their velocities, under different pressures or perpendicular heights, are as the square roots of those pres-

* It makes no difference whether the water stands perpendicularly, or inclined, above the aperture, [see Plate III. fig. 22.] provided the perpendicular height be the same; or whether the quantity be great or small, provided it be sufficient to keep the fluid up to the same height.

sures or heights, and their perpendicular heights or pressures, are as the squares of their velocities.*

3. Their quantities expended through equal apertures, in equal times, under unequal pressures, are as their velocities simply.†

4. Their pressures or heights being the same, their effects are as their quantities expended.‡

5. Their quantities expended being the same, their effects are as their pressure, or height of their head directly.§

6. Their instant forces with equal apertures, are as the squares of their velocities, or as the height of their heads directly.

7. Their effects are as their quantities, multiplied into the squares of their velocities.‖ See Art. 46.

8. Therefore, their effects or powers with equal apertures are as the cubes of their velocities.¶

* This law is similar to the 4th law of falling bodies, their velocities being as the square root of their spaces passed through; and by experiment it is known, that water will spout from under a 4 feet head, with a velocity of 16,2 feet, per second, and from under a 16 feet head, 32,4 feet per second, which is only double to that of a 4 feet head, although there be a quadruple pressure. Therefore, by this law, we can find the velocity of water spouting from under any given head: for as the square root of 4 equal 2, is to 16,2 its velocity, so is the square root of 16 equal 4, to 32,4 squared, to 16, its head: by which ratio we can find the head that will produce any velocity.

† It is evident, that a double velocity will vent a double quantity.

‡ If the pressure be equal, the velocity must be equal; and it is evident, that double quantity, with equal velocity, will produce a double effect.

§ That is, if we suppose 16 cubic feet of water to issue from under a 4 feet head in a second, and an equal quantity to issue in the same time from under a 16 feet head, then their effects will be as 4 to 16. But we must note, that the aperture in the last case, must be only half of that in the first, as the velocity will be double.

‖ This is evident, from this consideration; namely: that a quadruple impulse is required to produce a double velocity, by law 2d, where the velocities are as the square roots of their heads: therefore, their effects must be as the squares of their velocities.

¶ The effects of striking fluids with equal apertures are as the cubes of their velocities, for the following reasons; namely: 1st. If an equal quantity strike with double velocity, the effect is quadruple on that account by the 7th law; and a double velocity expends a double quantity by 3d law; therefore, the effect is augmented to the cube of the velocity.—The theory for undershot wheels agrees with this law also.

9. Their velocity, under any head, is equal to the velocity that a heavy body would acquire in falling from the same height.*

10. Their velocity is such, under any head or height, as will pass over a distance equal to twice the height of the head, in a horizontal direction, in the time that a heavy body falls the distance of the height of the head.

11. Their action and re-action are equal.†

12. They being non-elastic, communicate only half their real force by impulse, in striking obstacles; but by their gravity produce effects, equal to elastic or solid bodies.‡

A SCALE

Founded on the 3d, 6th, and 7th laws, showing the effects of striking Fluids with different Velocities.

Aperture.	Multiplied by the	Velocity.	Is equal the	Quantity expended.	Which multiplied by the	Square of the velocity,	Is equal the	Effect.	Which is as the	Cubes of the velocity.
1	×	1	=	1	×	1	=	1	as	1
1	×	2	=	2	×	4	=	8	as	8
1	×	3	=	3	×	9	=	27	as	27
1	×	4	=	4	×	16	=	64	as	64

* The falling body is acted on by the whole force of its own gravity, in the whole of its descent through any space; and the whole sum of this action that is acquired as it arrives at the lowest point of its fall is equal to the pressure of the whole head or perpendicular height above the issue; therefore their velocities are equal.

† That is, a fluid re-acts back against the penstock with the same force that it issues against the obstacle it strikes; this is the principle by which Barker's mill, and all those that are denominated improvements thereon, move.

‡ When non-elastic bodies strike an obstacle, one half of their force is spent in a lateral direction, in changing their figure, or in splashing about. See Art. 9.

For want of due consideration or knowledge of this principle, many have been the errors committed by applying water to act by impulse, when it would have produced a double effect by its gravity.

ARTICLE 46.

DEMONSTRATION OF THE 7TH LAW OF SPOUTING FLUIDS.

Let A F, (plate III. fig. 26,) represent a head of water 16 feet high, and suppose it divided into 4 different heads of 4 feet each, as B C D E; then suppose we draw a gate of 1 foot square at each head successively, always sinking the water in the head, so that it will be but 4 feet above the centre of the gate in each case.

Now it is known that the velocity under a 4 feet head, is 16,2 feet per second; to avoid fractions say 16 feet, which will issue 16 cubic feet of water per second, and for sake of round numbers, let unity or 1 represent the quantity of a cubic foot of water; then, by the 7th law the effect will be as the quantity multiplied by the square of the velocity; that is, 16 multiplied by 16 is equal to 256, which, multiplied by 16, the quantity, is equal to 4096, the effect of each 4 feet head; and 4096 multiplied by 4 is equal to 16384, for the sum of effects of all the 4 feet heads.

Then, as the velocity under a 16 feet head is 32,4 feet, to avoid fractions say 32, the gate must be draw to only half the size, to vent the 16 cubic feet of water per second as before (because the velocity is double;) then to find the effect, 32 multiplied by 32 is equal to 1024; which multiplied by 16, the quantity, gives the effect 16384, equal the sum of all the 4 feet heads, which agrees with the practice and experience of the best teachers. But if their effects were as their velocities simply, then the effect of each 4 feet head would be, 16 multiplied by 16, equal to 256; which, multiplied by 4, is equal to 1024, for the sum of the effects of all the 4 feet heads; and 16 multiplied by 32 equal to 512, for the effect of the 16 feet head, which is only half of the effect of the same head when divided into 4 parts; which is contrary to both experiment and reason.

Again, let us suppose the body A of quantity 16, to be perfectly elastic, to fall 16 feet and strike F, a perfectly elastic plane, it will (by laws of falling bodies) strike with a velocity of 32 feet per second, and rise 16 feet to A again.

But if it fall only to B, 4 feet, it will strike with a velocity of 16 feet per second, and rise 4 feet to A again. Here the effect of the 16 feet fall is 4 times the effect of the 4 feet fall, because the body rises 4 times the height.

But if we count the effective momentum of their strokes to be as their velocities simply, then 16 multiplied by 32 is equal to 512, the momentum of the 16 feet fall; and 16 multiplied by 16 is equal to 256; which, multiplied by 4, is equal to 1024, for the sum of the momentums of the strokes of 16 feet divided into 4 equal falls, which is absurd. But if we count their momentums to be as the squares of their velocities, the effects will be equal.

Again, it is evident that whatever impulse or force is required to give a body velocity, the same force or resistance will be required to stop it; therefore, if the impulse be as the square of the velocity produced, the force or resistance will be as the squares of the velocity also. But the impulse is as the squares of the velocity produced, which is evident from this consideration: Suppose we place a light body at the gate B, of 4 feet head, and pressed with 4 feet of water; when the gate is drawn it will fly off with a velocity of 16 feet per second; and if we increase the head to 16 feet, it will fly off with 32 feet per second. Then, as the square of 16 equal to 256 is to the square of 32 equal to 1024, so is 4 to 16. Q. E. D.

ARTICLE 47.

THE 7TH LAW IS IN ACCORDANCE WITH PRACTICE.

Let us compare this 7th law with the theory of undershot mills, established Art. 41, where it is shown that the power is to the effect as 3 to 1. By the 7th law, the quantity shown by the scale, Plate II. to be 32,4 multiplied by 1049,76 the square of the velocity, which is equal to 3401,2124, the effect of the 16 feet head; then, for the effect of a 4 feet head, with equal apertures, quantity by scale 16,2, multiplied by 262,44, the velocity squared, is equal to 425,15 28, the effect of a 4 feet head; here the ratio of the effect is as 8 to 1.

Then, by the theory, which shows that an undershot wheel will raise 1-3d of the water that turns it, to the whole height from which it descended, the 1-3d of 32,4 the quantity, being equal to 10,8, multiplied by 16, perpendicular ascent, which is equal to 172,8, effect of a 16 feet head: and 1-3d of 16,2 quantity, which is equal to 5,4 multiplied by 4, perpendicular ascent, is equal to 21,6 effect of a 4 feet head, by the theory: and here again the ratio of the effects is as 8 to 1; and,

as 3401,2124, the effect of 16 feet head, } by 7th law,
is to 425,1825, the effect of a 4 feet head, }

so is 172,8, the effect of 6 feet head, } by the theory
to 21,6, the effect of 4 feet head, }

The quantities being equal, their effects are as the height of their heads directly, as by 5th law, and as the squares of their velocities, as by the 7th law. Hence it appears, that the theory agrees with the established laws.

Application of the Laws of Motion to Undershot Wheels.

To give a short and comprehensive detail of the ideas I have collected from different authors, and from the result of my own reasoning on the laws of motion and of spouting fluids, as they apply to move undershot mills, I refer to fig. 44, Plate V.

Let us suppose two large wheels, one of 12 feet, and the other of 24 feet radius, the circumference of the largest will then be double that of the smallest: and let A 16, and C 16, be two penstocks of water, of 16 feet head each, then,—

1. If we open a gate of 1 square foot at 4, to admit water from the penstock A 16, to impinge on the small wheel at I, the water being pressed by 4 feet head, will move 16 feet per second (we omit fractions.) The instant pressure or force on that gate, being four cubic feet of water, it will require a resistance of 4 cubic feet of water from the head C 16 to stop it, and hold it in equilibrio, (but we suppose the water cannot escape, unless the wheel moves, so that no force be lost by non-elasticity.) Here equal quantities of matter, with equal velocities, have their momentums equal.

2. Again, suppose we open a gate of 1 square foot at A 16 under 16 feet head, it will strike the large wheel at k, with velocity 32, its instant force or pressure being 16 cubic feet of water, it will require 16 cubic feet resistance, from the head C 16, to stop or balance it. In this case, the pressure, or instant force, is quadruple to the first, and so is the resistance, but the velocity only double. In these two cases the forces and resistances being equal quantities, with equal velocities, their momentums are equal.

3. Again, suppose the head C 16 to be raised to E, 16 feet above 4, and a gate drawn 1-4th of a square foot, then the instant pressure on the float I of the small wheel, will be 4 cubic feet, pressing on 1-4th of a square foot, and will exactly balance 4 cubic feet, pressing on 1 square foot from the head A 16; and the wheel will be in equilibrio, (supposing the water cannot escape until the wheel moves as before,) although the one has power of velocity 32, and the other only 16, feet per second; their loads at equilibrio are equal, consequently, their loads at a maximum velocity and charge will be equal, but their velocities different.

Then, to try their effects, suppose, first, the wheel to move by the 4 feet head, its maximum velocity to be half the velocity of the water, which is 16, and its maximum load to be half its greatest load, which is 4, by Waring's theory; then the velocity 16÷2 multiplied by the load 4÷2=16, the effect of the 4 feet head, with 16 cubic feet expended; because the velocity of the water is 16, and the gate 1 foot.

Again, suppose it to move by the 16 feet head and gate of 1-4th of a foot; then the velocity 32÷2 multiplied by the load 4÷2 = 32, the effect, with but 8 cubic feet expended, because, the velocity of the water is 32, and the gate but 1-4th of a foot.

In this case the instant forces are equal, each being 4; but the one moving a body only 1-4th as heavy as the other, moves with velocity 32, and produces effect 32, while the other, moving with velocity 16, produces effect 16. A double velocity, with equal instant pressure, produces a double effect, which seems to be according to the Newtonian theory. And in this sense the momentums of bodies in motion are as their quantities, multiplied into their simple velocities, and this is what I call the instant momentums.

But when we consider, that in the above case it was the quantity of matter put in motion, or water expended, that produced the effect, we find that the quantity 16, with velocity 16, produced effect 16; while quantity 8, with velocity 32, produced effect 32. Here the effects are as their quantities, multiplied into the squares of their velocities; and this I call the effective momentums.

Again, if the quantity expended under each head had been equal, their effects would have been 16 and 64, which is as the squares of their velocities, 16 and 32.

4. Again, suppose both wheels to be on one shaft, and let a gate of 1-8th of a square foot be drawn at 16 C, to strike the wheel at K, the head being 16 feet, the instant pressure on the gate will be 2 cubic feet of water, which is half of the 4 feet head with 1 foot gate, from A 4 striking at I; but the 16 feet head, with instant pressure 2, acting on the great wheel, will balance 4 feet on the small one, because the lever is of double length, and the wheels will be in equilibrio. Then, by Waring's theory, the greatest load of the 16 feet head being 2, its load at a maximum will be 1, and the velocity of the water being 32, the maximum velocity of the wheel will be

16. Now the velocity $16\times1=16$, the effect of the 16 feet head; and gate of 1-8th of a foot, the greatest load of the 4 feet head being 4, its maximum load 2, the velocity of the water 16, and the velocity of the wheel 8: now $8\times2=16$, the effect. Here the effects are equal, and here, again, the effects are as the instant pressures, multiplied into their simple velocities: and the resistances that would instantly stop them must be equal thereto, in the same ratio.

But when we consider, that in this case the 4 feet head expended 16 cubic feet of water, with velocity 16, and produced effect 16; while the 15 feet head expended only 4 cubic feet of water, with velocity 32, and produced effect 16, we find that the effects are as their quantities, multiplied into the squares of their velocities.

And when we consider, that the gate of 1-8th of a square foot, with velocity 32, produced effects equal to the gate of 1 square foot, with velocity 16, it is evident, that if we make the gates equal, the effects will be as 8 to 1; that is, the effects of spouting fluids, with equal apertures, are as the cubes of their velocities; because, their instant forces are as the squares of their velocities, by 6th law, although the instant forces of solids are as their velocities simply, and their effects as the squares of their velocities, a double velocity does not double the quantity of a solid body to strike in the same time.

ARTICLE 48.

THE HYDROSTATIC PARADOX.

The pressure of fluids is as their perpendicular heights, without any regard to their quantity: and their pressure upwards is equal to their pressure downwards. In short, their pressure is every way equal, at any equal distance from their surface.

In a vessel of cubic form, whose sides and bottom are equal, the pressure on each side is just half the pressure on the bottom; therefore, the pressure on the bottom and sides is equal to three times the pressure on the bottom.