CHAPTER XIII.

KINEMATIC CLASSIFICATION OF MECHANISMS.

101. Historical Sketch. — In treating of the theory of Mechanisms, it has been the aim of many writers to devise some method of analysis whereby mechanical contrivances in general might be resolved into their several component parts, capable of being represented, if necessary, by symbols, and capable also of being recombined in such a fashion as to produce new mechanisms. Such a system, if complete and workable, would of course be of great service to the inventor, and would save him from the fate, only too common, of designing with great toil some device which has been known and used for years. In the words of Willis, "there appears no reason why the construction of a machine for a given purpose should not, like any usual problem, be so reduced to the dominion of the mathematician as to enable him to obtain, by direct and certain methods, all the forms and arrangements that are applicable to the desired purpose, from which he may select at pleasure." It must be confessed that so far no such system of analysis and synthesis has been found of any great practical value; many of the proposals, however, are interesting and suggestive, and a brief account of some of them will not be out of place in this book. Before entering upon it we may glance at the historical development of the subject of the Kinematics of Machines.

A book dating from the eighteenth century * seems to be the first treatise on machines which can be considered at

* Leupold. Theatrum Machinarum. 1724.

all systematic. Leupold's predecessors had indeed described sundry machines and devices, but their order of arrangement was always arbitrary, and no attempt was made to study machines by considering the relative motions of their parts. The theory of machines, treated either from the kinematic or dynamic standpoint, did not in fact exist.

Euler * taught that the motions of rigid bodies should be investigated by the methods of geometry, as well as by the aid of dynamics, but it does not appear that he had in view the special application of these principles to the motions of the parts of machines. Monge in 1794 conceived the idea of treating machines as contrivances for changing one kind of motion into another, and was the first to suggest that the essential "elements of machines" should be enumerated and studied. His system formed the basis of the course adopted in the Ecole Polytechnique soon after its foundation—a course laid out by Lanz and Bétancourt,† and classifying the motions of the parts of machines as (1) rectilinear, (2) circular, or (3) curvilinear. Combinations of these motions are considered, while each motion may be continuous or alternate. The work of Lanz and Bétancourt was incomplete, because no attempt was made to calculate these various motions; their scheme underwent many modifications, and has not survived. A system somewhat similar in intention, but differing in detail, was propounded by Borgnis.‡ It has met with the same fate.

It is to the physicist Ampère § that we owe an important advance. He saw clearly that a mechanism should be studied as "an instrument by the help of which the direction and velocity of a given motion can be altered"; thus going further than Euler, and laying the foundations of that science of Machines to which, in accordance with his suggestion, we apply the name Kinematics.

* Euler. Theoria Motus Corporum. 1765.

† Lanz and Bétancourt. Essai sur la composition des Machines. 1808.

‡ Borgnis. Traité complet de Mécanique appliquée aux Arts. 1818.

§ Ampère. Essai sur la philosophie des Sciences. 1834.

Ampère was followed by Willis,* who confined himself to the consideration of what he termed the "Elements of Pure Mechanisms," and did not deal with the "generalities of motion." The "Principles of Mechanism" takes a less abstract view of the science of Kinematics than Ampère seems to have held, and in that book the author endeavors to form a system embracing all the elementary combinations of mechanism, and admitting of an investigation of their modifications of motion. He does not attempt to deal with dynamical questions, but gives practical and useful solutions of many leading problems in applied kinematics. His system of classification will receive some consideration in a later section; we shall see that its groundwork is the mode in which the motion is transmitted, or, as we should now express it, the kind of relative motion existing, in various mechanisms.

In several of his books Rankine † deals with kinematical questions, treated under such titles as the Geometry of Machinery and the Theory of Mechanism. His views were in some few respects erroneous and incomplete, and his nomenclature has not been followed to any large extent, but his system of dealing with the motion of machine parts by the aid of instantaneous centres, and his methods of solving certain special problems, were in many cases far more powerful and effective than any previously employed.

The appearance in collected form of the kinematical writings of Reuleaux ‡ furnished students with the first text-book whose methods have met with really wide acceptance. It is to Reuleaux that we owe the idea of a mechanism regarded as a chain made up of links any one of which may be considered as being fixed. Starting with this con-

* Willis. Principles of Mechanism. 1841. (Second Edition 1870.)

† Rankine. Applied Mechanics. 1858.
Manual of Machinery and Millwork. 1869.

‡ Reuleaux. Theoretische Kinematik. English Translation by Dr. Kennedy. 1876.

ception, and taking account of the relative motion of these links as determined by the pairing of their elements, we are led to a wide and comprehensive view of the whole kinematic theory of mechanisms. The earlier work of Reuleaux has now been supplemented by the publication of a second part of his text-book.*

Burmester's important treatise † is not so well known to English-speaking readers as it should be. Only the first volume, dealing with plane motion, has yet been published. Burmester's method of treatment differs from that of Reuleaux in making a more liberal use of purely mathematical and geometrical principles, but the two authors agree in their fundamental conception of the subject, and, to a large extent, in their nomenclature and definitions. A considerable amount of space is devoted by Burmester to the kinematics of a plane rigid system; he deals with the principles of constraint in plane motion, and passes on to the consideration of plane mechanisms and the relative displacement, velocity, and acceleration of their various parts. The second volume is to treat, after a similar fashion, of non-plane motion.

102. Classification of Willis. Babbage's Notation. — The following sections contain a short account of some of the schemes suggested for classifying and symbolizing the various kinds of mechanisms.

Like almost all his predecessors, Willis contented himself with proposing a scheme of classification without endeavoring to invent any notation, or system of signs, by which a given mechanism could be represented by a formula. Without apparent reason, Willis excludes from his system all hydraulic machines. Some other classes of mechanism, for example those including springs, are also omitted. In fact he considers as "pure mechanisms" only certain types

* Reuleaux. Die praktischen Beziehungen der Kinematik zu Geometrie und Mechanik. 1900.

† Burmester. Lehrbuch der Kinematik, 1888.

of machines, which seem to have been selected in a somewhat arbitrary fashion. In these machines, according to Willis, motion is transmitted in "elementary combinations" by five methods, namely:

Division.	Method of Transmission.	Example.
A.	By rolling contact.	Toothed gearing of various sorts.
B.	By sliding contact.	Cams, screws, worm- and screw-gearing, escapements.
C.	By wrapping connection.	Bands, chains, and other gearing.
D.	By linkwork.	Cranks, eccentrics, and other linkwork. Ratchet-wheels and clicks.
E.	By reduplication.	Tackle of all sorts.

Each of these five main divisions is again separated into three classes, in which the velocity ratio is either (*a*) constant, (*b*) varying, and (*c*) constant or varying; while due regard is had to the question whether the "directional relation" is constant of varying.

This system or classification has not been widely used, and possesses certain manifest imperfections. It was, however, a great advance on that of Lanz and Bétancourt or on that of Borgnis, because it was designed with a view of facilitating calculations regarding the relative motions, or velocity ratios, in mechanisms, rather than with the aim of classifying mechanisms for purely descriptive purposes.

In the "Principles of Mechanism" Willis devotes some space to the exposition of the scheme of notation proposed by Babbage; * a scheme devised by that ingenious inventor primarily for the purpose of clearly representing the relations of the parts of his calculating-machine, and especially

* A Method of Expressing by Signs the Action of Machinery. Phil. Trans., 1826.

applicable to complex trains of wheel and ratchet gearing. As this notation involves the construction of an elaborate sheet or diagram for each machine, it by no means answers the purpose of a system such as that of Reuleaux, which will be described later, where each mechanism is to be denoted by a formula of three or more symbols. Babbage's method of notation corresponds more closely to that employed by clock and watch-makers, in which the various wheels are represented by the numbers of their teeth, written in successive lines, placing vertically over each other the numbers of wheels which gear together. Thus

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48
 6 - 45
      6 - 30
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would represent a wheel-train comprising a "great wheel" of 48 teeth gearing with a pinion of 6 teeth, the pinion-arbor or axis carrying a second wheel of 45 teeth, gearing in its turn with a 6-tooth pinion whose arbor carries an escape-wheel of 30 teeth. Babbage, however, shows on his diagram the kind of motion, whether uniform, intermittent, variable, or continuous, of each part with relation to the frame of the machine, and Willis gives an interesting example * of such a diagram, as constructed for a sawmill. It would appear that Babbage's notation, while extremely convenient in certain cases, by no means answers the purpose of a general scheme by means of which the mode of action and relative motions in any given mechanism may be indicated.

103. Classification and Notation of Reuleaux. — Such a system has been devised by Reuleaux,† and is explained and used in his text-book. It is intended to be perfectly general in its application, and includes signs of three kinds, which denote (1) the class or name of the body or link referred

* Principles of Mechanism, Ed. 1870, p. 288.

† Kinematics of Machinery, English Ed., p. 251.

to, as distinguished by its geometrical shape or its nature; (2) the form of the body, whether solid or full, or hollow or open, whether plane or curved; and lastly, (3) the relation of one element to its companion, or of one link to the next in the chain. Some special symbols are required to indicate incomplete pairs, methods of closure, and so on.

In the first division the following *name* symbols have been adopted:

S	Screw.	G	Sphere or globe.
R	Solid of revolution.	A	Sector or arc.
P	Prism.	Z	Tooth or projection.
C	Cylinder.	V	Vessel or chamber.
K	Cone.	T	Tension-organ.
H	Hyperboloid.	Q	Pressure-organ.

These symbols require no explanation.

With regard to the next kind of symbols, those of *form*, it is evidently necessary to indicate among other particulars whether a given body is full, open, or plane; whether its profile is curved or non-circular, or has upon it teeth; or whether its profile or section is prismatic. A link, as we have seen, may be liquid or gaseous, and a large number of other cases may be suggested, all of which should be covered by any general system of symbols. Reuleaux proposes to do this by adding to the Roman capital letters which he selects as the name-symbols, certain form-symbols, written to the right of the name-symbol, and either above or below it. A few examples will illustrate the way of doing this. We may use the following:

+	full or solid.	°	plane.
–	open or hollow.	~	curved profile.

From these and the preceding symbols we have, among many others:

C^+	full cylinder.	C^-	open cylinder.
S^+	screw or bolt.	S^-	nut.

$\tilde{P}-$ hollow or open prism whose base has a curved outline.
$\tilde{C}_z^+$ non-circular spur-wheel with external teeth.
K_z° face-wheel (plane bevel-wheel).
T_p prismatic tension-organ (flat belt).
T_c circular tension-organ (wire).
Q_λ liquid pressure-organ.
Q_γ gas, air, steam, etc.
V^- cylinder of an engine or pump.
V^+ the piston working in it.

The third class of symbols express relation, as regards pairing, linkage, or fixing, or as regards position and magnitude. Pairing is denoted by a comma, linkage by a dot or dotted line; a fixed link is indicated by underlining, and the usual signs are employed for equality, parallelism, and so on. For example:

$C^- \ldots C^-$ link connecting two open cylinders or eyes.
P_z, C_z^+ rack pairing with a pinion.
$S^+ = S^-$ screw pair, screw and nut being of course equal in size.

Incompleteness is indicated by the use of the sign of division, so that we get:

$\frac{C^-}{2}$ portion of an open cylinder.

$\frac{C^+}{f}$ a full cylinder paired by force-closure.

The method of closure is indicated by the divisor.

As an example of the method of writing out the formula for a simple mechanism, we may refer to Fig. 133. The spur-wheel mechanism *acd* would be written (*d* being the fixed link):

$$\underbrace{C^+ \ldots | \ldots C_z^+}, \ \underbrace{C_z^+ \ldots | \ldots C^+} \ = \ \underbrace{\underline{C^- \ldots \| \ldots C^-_=}}$$

(referring to link *a*, link *b*, and link *d*).

Here | means con-axial; ‖ means parallel.

After describing and enumerating the various symbols, Reuleaux proceeds to show how the resulting formulæ may be shortened. He employs (S), (C), and (P) for a screw pair, a turning pair, and a sliding pair, respectively, and would write $(C_4^{\|})\frac{d}{a}$ for "a chain formed of four links, each connecting two parallel cylindric elements"; *d* being the fixed link and *a* the one which drives the chain. This is of course the quadric cylindric crank chain. His symbols have a very wide range of adaptability; the reader will be interested, for example, in the formula for a paddle-steamer, which is

$$C^{+} \ldots | \ldots C_z^{+}, Q_\lambda \ldots Q_\lambda, V^{-} \ \ldots + \ldots C_{=}^{-}$$

This may be contracted to $(C^{i}\, C_{z\lambda} V_\lambda)\frac{b}{a}$.

Here *b* is the liquid link, *a* the paddle-wheels, and *c* the ship itself. V_λ is a contraction for V^{-}, Q_λ, and $C_{z\lambda}$ for C_z, Q_λ; + is the sign for "crossed at right angles" when used as the symbol of relation of the elements of a link.

The original text-book must of course be consulted if any real acquaintance with the scheme is desired; the examples given here will serve to indicate the scope and posibilities of the system.

104. Classification of Hearson.—The most recent system of notation devised by Professor Hearson * differs essentially from that of Reuleaux, for it is based on a somewhat different conception of the meaning of the term *machine*. Hearson considers that a machine is to be regarded as "an embodiment of a combination of elementary motions (of which it will be found that the number of kinds is comparatively limited)"; these "elementary motions" being the relative motions of the machine-parts. He treats first of plane mechanisms, and suggests the following symbols:

* Phil. Trans., 1896, Vol 187, p. 15.

O a motion of complete and continuous relative rotation.
U a swinging to-and-fro motion, like that of a pendulum, consisting of successive partial rotations in opposite senses.
I a sliding motion.

Taking a four-link mechanism as the general case (three and five or more links being inadmissible in simple machines for reasons already given), it is shown that there may be in such a mechanism either

four O motions,
or three O's and one U } under certain conditions as
or two O's and two U's, } to length of links,
or four U motions;

while it is impossible to have one O motion only and three U's.

On considering the substitution of I motions for O's and U's, it is found that (in all) fourteen combinations of O's, U's, and I's are permissible.

In order to denote such motions as that of the teeth of a pair of spur-wheels, Hearson assigns the symbols

W for a combination of two U motions;
∞ for a combination of two O motions;
C for the wrapping motion of a belt on its pulley.

He further proposes to distinguish between the sense of motions by the use of large and small letters, so that, for example, two pulleys mounted on a frame joined by a crossed belt would be OcCO or oCco; if an open belt were employed the formula would become OcCo or oCcO.

Passing to spherical mechanisms, a similar system is outlined; certain limitations, however, are imposed by the differences between plane and spherical geometrical relations.

Adopting the symbol V for helical motion having a constant pitch ratio, and H for one in which the pitch ratio varies, it is found that we may have the mechanisms UVI, VVI, VVU, and VVV, of which there are eight inversions in all. With H motions four combinations, with eleven inversions, give us UHI, VHI, VHU, and VHV; these may be classed under the head of cylindrical mechanisms. Hearson proposes to group all remaining simple mechanisms in a fourth division, comprising those in which the axes neither meet nor are parallel. He then discusses compound mechanisms in which there are more than four separately moving pieces, and yet the motions are of a determinate character. This leads to a method of formulating such mechanisms; it will be sufficient here to give as an example the formula for the portion of a locomotive consisting of frame, piston and rod, connecting-rod, crank-axle, coupling-rod, and the crank of a second coupled driving-wheel. This is

$$\mathrm{O} - \mathrm{o} - \mathrm{I} - \mathrm{U} - \mathrm{o} - \mathrm{O}.$$

To explain this formula, it is to be noted that the link shown by the thick line is the frame or the link which is regarded as being fixed. The links which move in contact with it are the piston and rod (U – I), the crank-axle (o – O), and the coupled driving-wheel and its axle (O – o). The connecting-rod will be denoted by (U – o), and the coupling-rod by (O – o), the frame being (o – I – O). Here the large letters refer to turning motions in which the angle is increasing, while small letters indicate those in which the angle is diminishing. The lines connecting the letters of the complete formula show which motions are possessed by the various links. Hearson's scheme does not appear to contemplate the inclusion of fluid links, and, as outlined in the paper cited, is by no means so complete as that of Reuleaux.

105. Remarks on Classification.—It is not surprising that, up to the present, no system of kinematic classification has proved so simple, and at the same time so wide in its scope, as to be generally accepted as an assistance both to the inventor and to the student of the theory of mechanisms. The nomenclature and classification of Reuleaux and the