# CHAPTER XII.

## SPHERIC MOTION.

**96. Spheric Motion in General.** — Spheric motion has already been defined in § 6, and it has been explained that in such motion any given point in the moving body remains on the surface of a sphere described about a certain fixed point as centre. Two bodies having relative spheric motion will therefore have this point as a common centre.

We can study the relative motion of two or more such bodies by imagining that they are cut by a sphere described about the common point as centre, and we can then consider the movement of these spheric sections exactly as we considered the motion of the plane sections or projections of bodies having plane motion. Plane motion may indeed be looked upon as a particular case of spheric motion where the radius of the sphere is infinitely great.

We may therefore suppose that propositions proved with regard to plane motion will hold good, with certain necessary modifications, with regard to spheric motion also. It will be convenient, first of all, to consider the motion of a spheric figure on the sphere of motion, just as we considered in § 5 the motion of a plane figure in the plane of motion. The position of the spheric figure will of course be defined if we know the position of two of its points.

In Fig. 206 ($a$) a figure on the surface of a sphere $LMN$ has the positions of two of its points ($A$ and $B$) defined. Let the figure, which represents a body having spheric motion, be moved from a position $AB$ to a new position, $A_1B_1$; the movement being executed in a very small period of time, and being therefore an exceedingly small displacement. The paths of the points $A$ and $B$ will then practically coincide with portions of great circles passing through $A$ and $A_1$, and $B$ and $B_1$, respectively. Now let arcs of great circles be drawn passing through $L$ and $M$, the middle points of $AA_1$ and $BB_1$; let the planes of these great circles be respectively perpendicular to those of the great circles $ALA_1$ and $BMB_1$, and let them intersect at $N$. Draw $ON$ passing

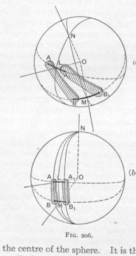


FIG. 206.

through $O$, the centre of the sphere. It is then evident that the actual small displacement of the body $AB$ is the same as if it had undergone a rotation about the axis $ON$, for $N$ is the point on the surface of the sphere at which $AA_1$ and $BB_1$ subtend equal spherical angles. This follows from the fact that the spherical triangles $ANB$ and $A_1NB_1$ are equal in all respects.

It may happen that $L$ and $M$ both lie on the same great circle, as in Fig. 206 ($b$), in which case our construction fails. The point $N$ is now to be taken at the intersection of the great circles $AB$ and $A_1B_1$, and it is evident, as before, that the angle subtended at $N$ by the arcs $AA_1$ and $BB_1$ is the same, and that $ON$ is the axis of rotation. Any actual motion of $AB$ on the surface of the sphere may be considered as being made up of a series of infinitely small displacements, to each of which there corresponds one position of the axis $ON$. $ON$ is therefore the *virtual axis* of the motion of $AB$ with regard to the sphere. The reader should compare the foregoing argument with that in § 5 applying to plane motion.

We may call the surface described by $ON$ in the sphere the axode of $AB$ with respect to the sphere. Two bodies, $a$ and $b$, having relative spheric motion will of course have a pair of such axodes; the axode of $a$ being imagined as being described in the body $b$ and *vice versa*, just as in the case of plane motion: and, further, this relative motion may be represented by the rolling together of such axodes.

Perhaps an example may make this clearer. Fig. 207 represents the pitch surfaces of a pair of bevel-wheels; their axes intersecting at $O$. These wheels are intended to transmit angular motion uniformly between shafts whose axes intersect, and their motion will evidently be exactly the same as that of a pair of circular cones of corresponding shape rolling together without slipping, and having a common apex at $O$. A pair of such cones, $a$ and $b$, and their frame, $c$, will have relative spheric motion about the point $O$. The lines $OA_{ac}$ and $OA_{bc}$ are of course the axes of $a$ and $b$ with respect to the frame $c$; the line $OA_{ab}$ along which the cones are in contact is the virtual axis of $a$ with respect to $b$, and the surface of each cone is therefore the axode of the other. (Compare the relation between the pitch surfaces in spur-gearing.)