CHAPTER VIII.

WHEEL-TRAINS AND MECHANISMS CONTAINING THEM. CAMS.

68. Simple and Compound Wheel-trains.—The determination of the velocity ratio in such a wheel-train as that of Fig. 130 involves no difficulty, for it is plain that one or

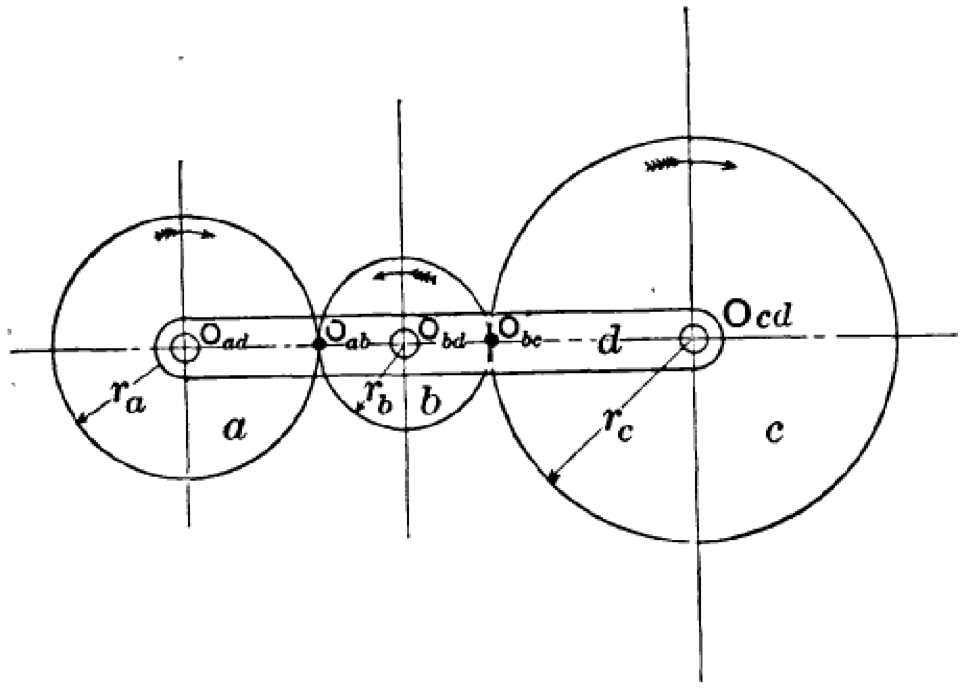

FIG. 130.

more intermediate wheels (as b) will not affect the numerical value of the velocity ratio of the first and last wheels. The linear velocity of the pitch-line of every wheel is the same, and the angular velocity ratio of the first and last, therefore, only depends on their own diameters, so that $\frac{\omega_{ad}}{\omega_{cd}} = \pm \frac{r_c}{r_a}$, the sign depending on the number of idle wheels. Intermediate or *idle* wheels thus simply reverse the direction of motion. When all the wheels in the train have external contact, the angular velocity ratio of the first wheel to the last has a positive value (or, both wheels turn in the same sense) if the number of axes is odd, while an even number of axes gives the velocity ratio a negative value. More complex wheel-trains, however, require further consideration. In Fig. 131 we have a compound spur-wheel mechanism of

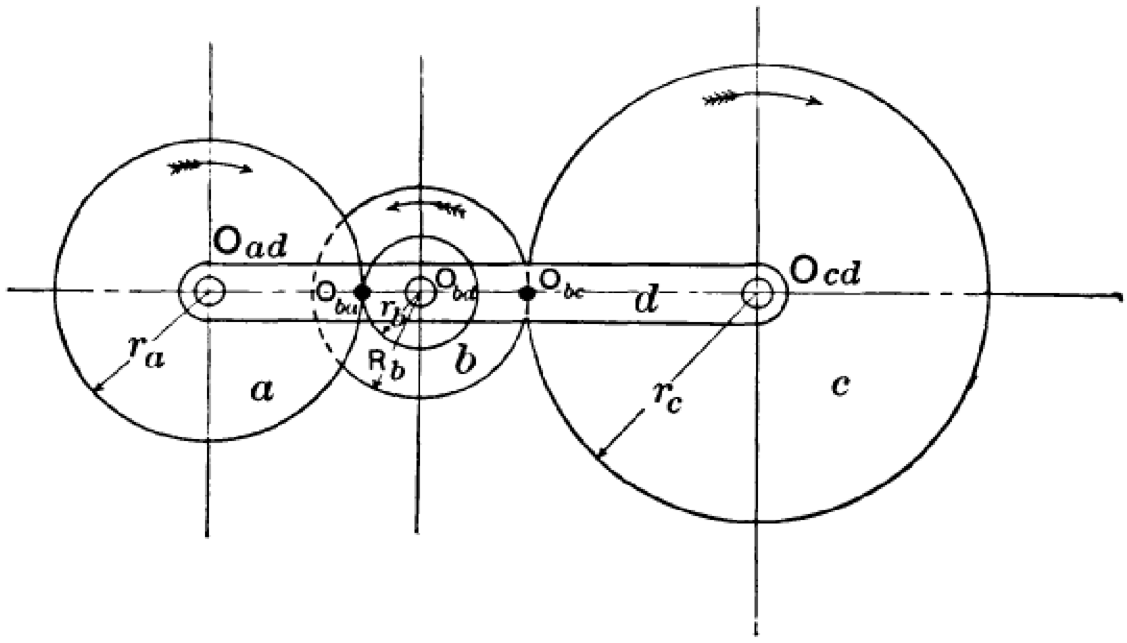

FIG. 131.

four links, *d* being fixed, while *b* consists of two wheels rigidly connected and turning on the same axis.

Let r_a, r_b, R_b, r_c, be the radii of the pitch-circles, then from § 64 we have

$$\frac{\omega_{ad}}{\omega_{bd}} = \frac{r_b}{r_a}.$$

Also,
$$\frac{\omega_{bd}}{\omega_{cd}} = \frac{r_c}{R_b}.$$

Hence
$$\frac{\omega_{ad}}{\omega_{cd}} = \frac{r_b \times r_c}{r_a \times R_b} = N.$$

Suppose *a* to be the driving-wheel, while *c* is the driven one; we see that the above result may be expressed by saying that

$$\text{velocity ratio} = \frac{\text{revolutions of driving-wheel}}{\text{revolutions of driven wheel}}$$
$$= \frac{\text{product of radii of followers}}{\text{product of radii of drivers}}.$$

Instead of radii we might evidently put numbers of teeth.

It would be easy to find a single pair of wheels having the same velocity ratio as the given train. For example, if we had a pair of wheels, A and C, such that

$$r_A - r_C = r_a + r_b + R_b + r_c \quad \text{and} \quad \frac{r_A}{r_C} = \frac{r_a \times R_b}{r_b \times r_c},$$

these would have the same velocity ratio and the same distance from centre to centre. The point of contact of their pitch-circles would divide the distance $O_{ad}\,O_{cd}$ externally in the proportion of the angular velocities of a and c, and would in fact be the point O_{ac}.* Hence in Fig. 131 we have only to divide the line of centres, graphically or otherwise, in the proper ratio to find the sixth virtual centre.

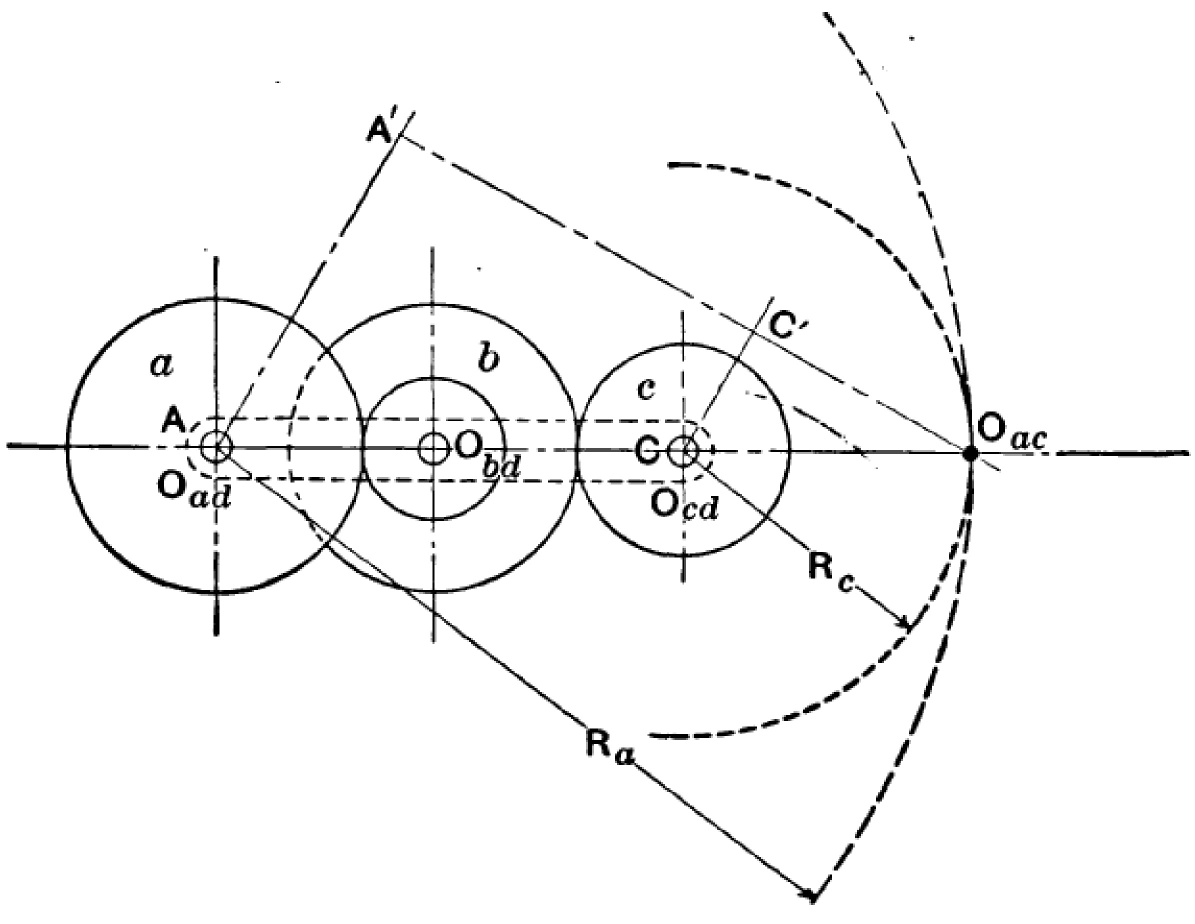

FIG. 132.

In doing this (as in working all problems connected with wheel trains), note must be taken of the sign of the velocity ratio, which depends on the presence or absence of *annular wheels* (i.e., wheels having internal contact), or of idle wheels, and also on the number of axes in the train. Take, for example, the two trains shown in Figs. 132 and 133, in

* For graphic methods of determining virtual centres of wheel-trains, see Kennedy, Mech. of Machinery, note to Chapter VI.

the first of which suppose $r_a=2$, $r_b=1$, $R_b=2$, $r_c=1.5$, so that the velocity ratio in Fig. 132 has the value

$$N=\frac{\omega_{ad}}{\omega_{cd}}=+\frac{1.5\times 1}{2\times 2}=+\frac{3}{8}.$$

In Fig. 133 we have a simple train having exactly the same numerical value for its velocity ratio (since $\frac{r_c}{r_a}=-\frac{3}{8}$), but in this case the negative value must be adopted, since the wheels a and c turn in opposite senses. In Fig. 132 O_{ac} may be found by drawing AA' CC' parallel to one another, and of lengths 8 and 3 respectively, to any convenient scale. The intersection of $A'C'$ with the line of centres fixes O_{ac}. Evidently the given train might be replaced by a pair of wheels of radii R_a and R_c, the larger being annular, having their centres at A and C, and their pitch-circles touching at O_{ac}, as shown by the dotted arcs. Again, in Fig. 133

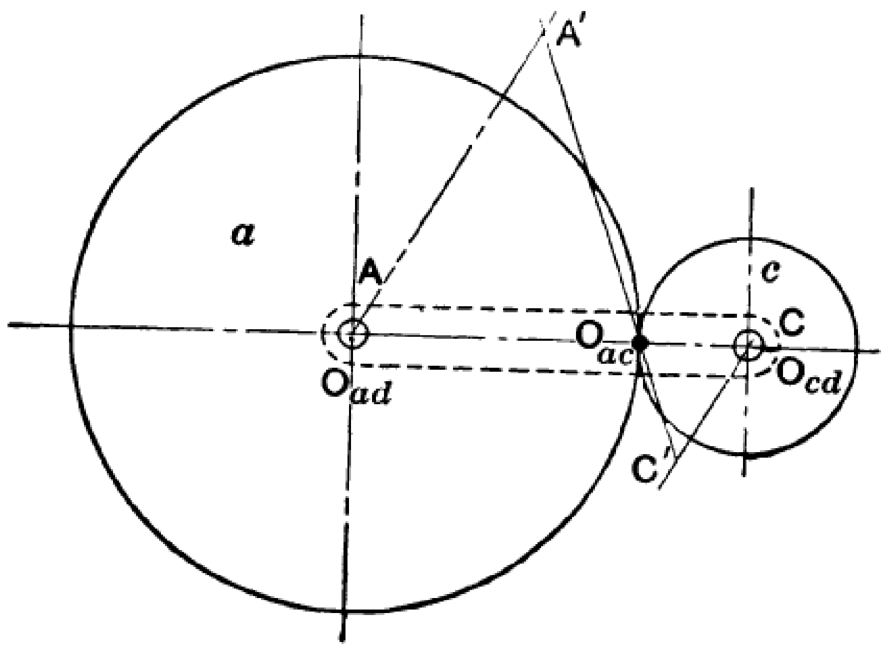

FIG. 133.

AA' CC' must be drawn parallel, but in opposite senses, so as to allow for the negative velocity ratio, and O_{ac} is, of course, the point of intersection of AC and $A'C'$.

It is by no means necessary to have the centres of all the wheels of a train in one straight line. The back-gear of a lathe, for example, is an instance of a compound *reverted train* in which the centres of the first and last wheels coincide.

This arrangement makes no difference in the numerical value of the velocity ratio, and is simply adopted for convenience in construction.

69. Epicyclic Gearing.—In the above examples of wheel-trains we have supposed the frame carrying the wheels to be the fixed link. Wheel gearing is often employed in which one of the wheels is the fixed link and the frame or arm carrying the remaining wheels is movable. Such gearing is called *epicyclic*, and we proceed to discuss some of its simpler cases.

We take first the mechanism of Fig. 133, but suppose a to be fixed, while d is rotated in a clockwise or positive sense (Fig. 134). Let N be the velocity ratio of the train, i.e., let

$$N = -\frac{r_a}{r_c} = \frac{\omega_{cd}}{\omega_{ad}}.$$

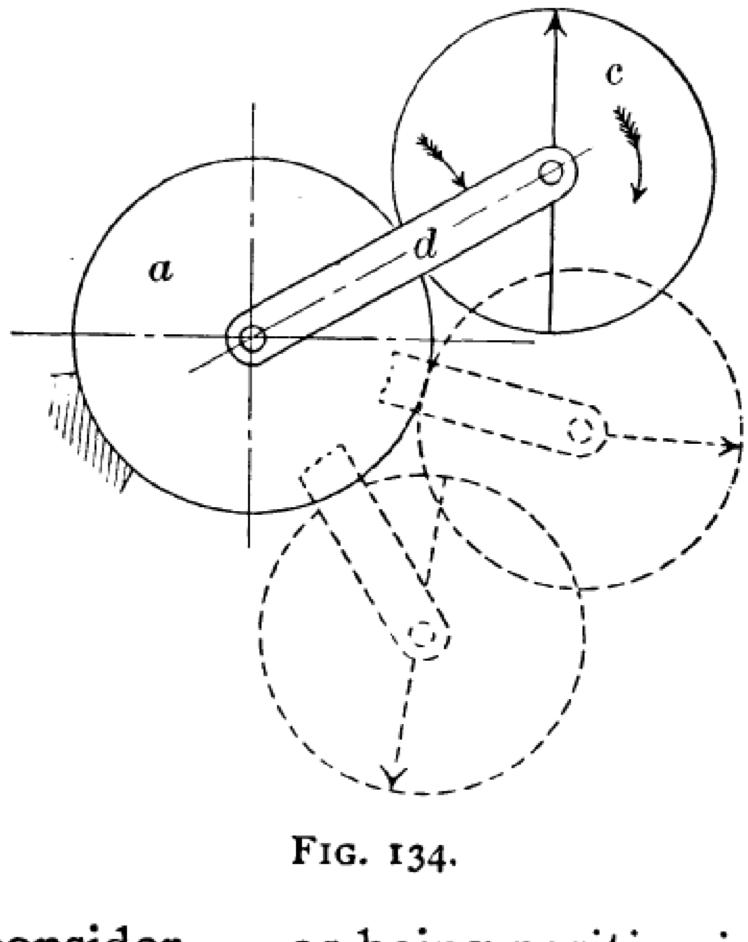

FIG. 134.

Plainly, if we consider ω_{da} as being positive in sign, then ω_{ad} must be negative, hence

$$\omega_{ad} = -\omega_{da}.$$

Now in any case where two bodies, c and d, have motion relatively to a third, a, which is fixed, any angular movement of c relatively to a may be looked on as the algebraic sum of the motions of c relatively to d and of d relatively to a. Thus

$$\omega_{ca} = \omega_{cd} + \omega_{da}$$
$$= +\omega_{cd} - \omega_{ad}$$
$$= -\omega_{ad}\left(1 + \frac{r_a}{r_c}\right),$$

or
$$\frac{\omega_{ca}}{\omega_{da}} = 1 - N,$$

where N is itself a negative quantity. A numerical example may, perhaps, make this clearer. Suppose the wheels a and c to have 100 teeth and 90 teeth respectively; these teeth have the same pitch, and we can, of course, take the ratio of the numbers of teeth instead of the ratio of the diameters or radii of the pitch-circles. Thus $\frac{r_a}{r_c} = \frac{10}{9}$; in other words, supposing d to be fixed, while a makes one revolution with

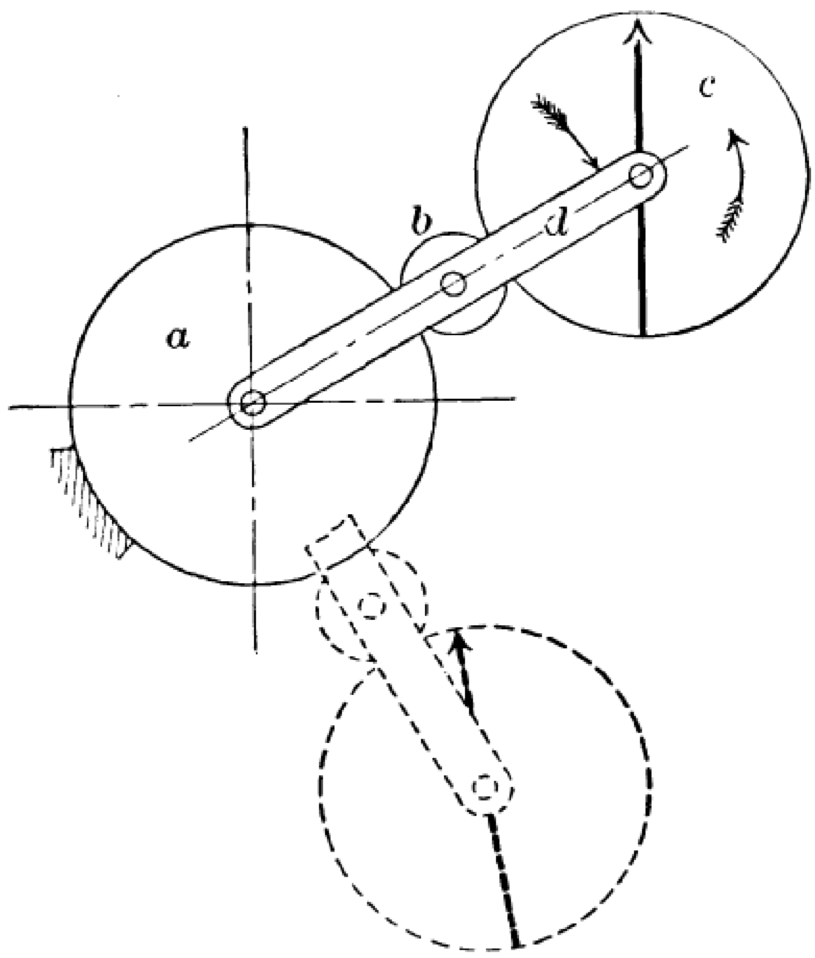

FIG. 135.

regard to d, c would make $1\frac{1}{9}$ in the opposite sense. Now suppose that in a certain time a makes -1 revolution, c making $+1\frac{1}{9}$, while d is at rest. Cause the whole mechanism to execute $+1$ rotation in the same time around O_{ad};

this brings a to rest, makes d perform $+1$ revolution, and therefore gives c $1\frac{1}{9}+1=2\frac{1}{9}$ revolutions in the same sense as that of the arm.

If an idle wheel, b, had been interposed between a and c, as in Fig. 135, we should have had $N=+\frac{r_a}{r_c}=\frac{\omega_{cd}}{\omega_{cd}}=$ a positive quantity, and $\omega_{ca}=\omega_{da}+\omega_{cd}$, whence $\frac{\omega_{ca}}{\omega_{da}}=1+\frac{\omega_{cd}}{\omega_{da}}=1-N$, as before. With the numbers of teeth, as in the example just given, and the train arranged as in Fig. 135, we should have, if $N=+1\frac{1}{9}$,

$$\frac{\omega_{ca}}{\omega_{da}}=1-1\frac{1}{9}=-\frac{1}{9};$$

i.e., for each revolution of the arm, c makes $\frac{1}{9}$ revolution in the reverse sense.

Fig. 136 represents a compound epicyclic reverted train. Let n_a, n_{b_1}, n_{b_2}, n_c be the numbers of teeth in the wheels

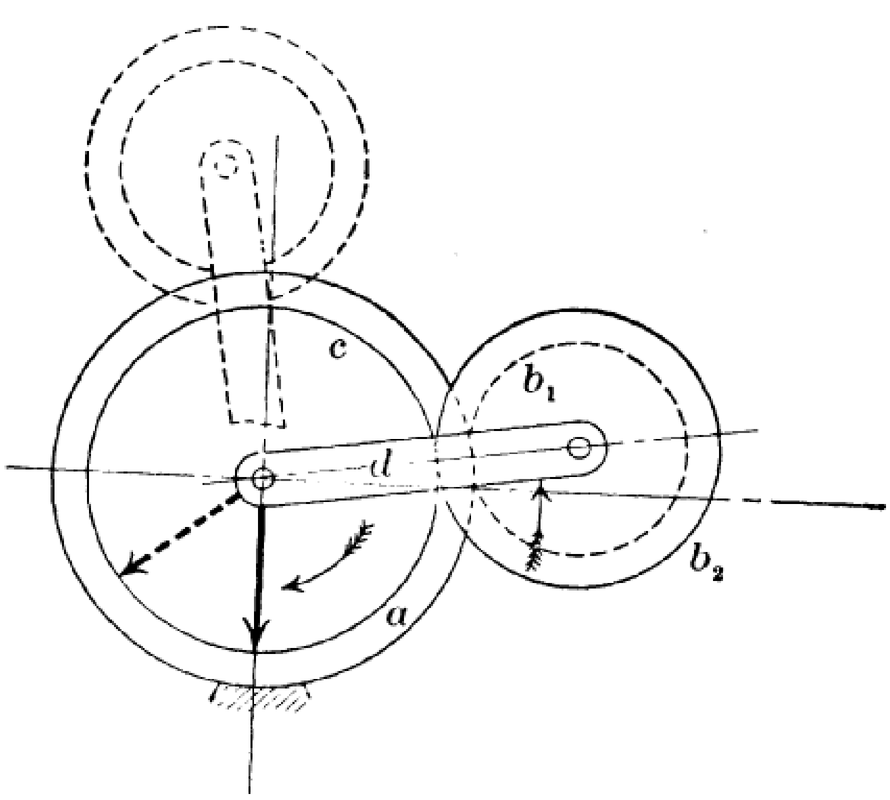

FIG. 136.

a, b_1, b_2, and c respectively. Evidently, if the pitch of both pairs is the same, $n_a+n_{b_1}=n_{b_2}+n_c$. The velocity ratio of the train will be positive and has the value

$$N=\frac{\omega_{cd}}{\omega_{ad}}=\frac{n_a\times n_{b_2}}{n_{b_1}\times n_c},$$

hence $$\omega_{cd} = -(N \times \omega_{da}).$$

Further, when a is the fixed link

$$\omega_{ca} = \omega_{cd} + \omega_{da} = \omega_{da}(1 - N).$$

Thus, for instance, suppose a has 30 teeth, b_1 has 15, and b_2 and c have respectively 20 and 25; then

$$N = +\frac{30 \times 20}{15 \times 25} = +1.6 = \frac{\omega_{cd}}{\omega_{ad}},$$

and $$\frac{\omega_{ca}}{\omega_{da}} = 1 - N = -0.6.$$

Thus c will make 0.6 revolution for each revolution of the arm, but in the opposite sense. Such a train might evi-

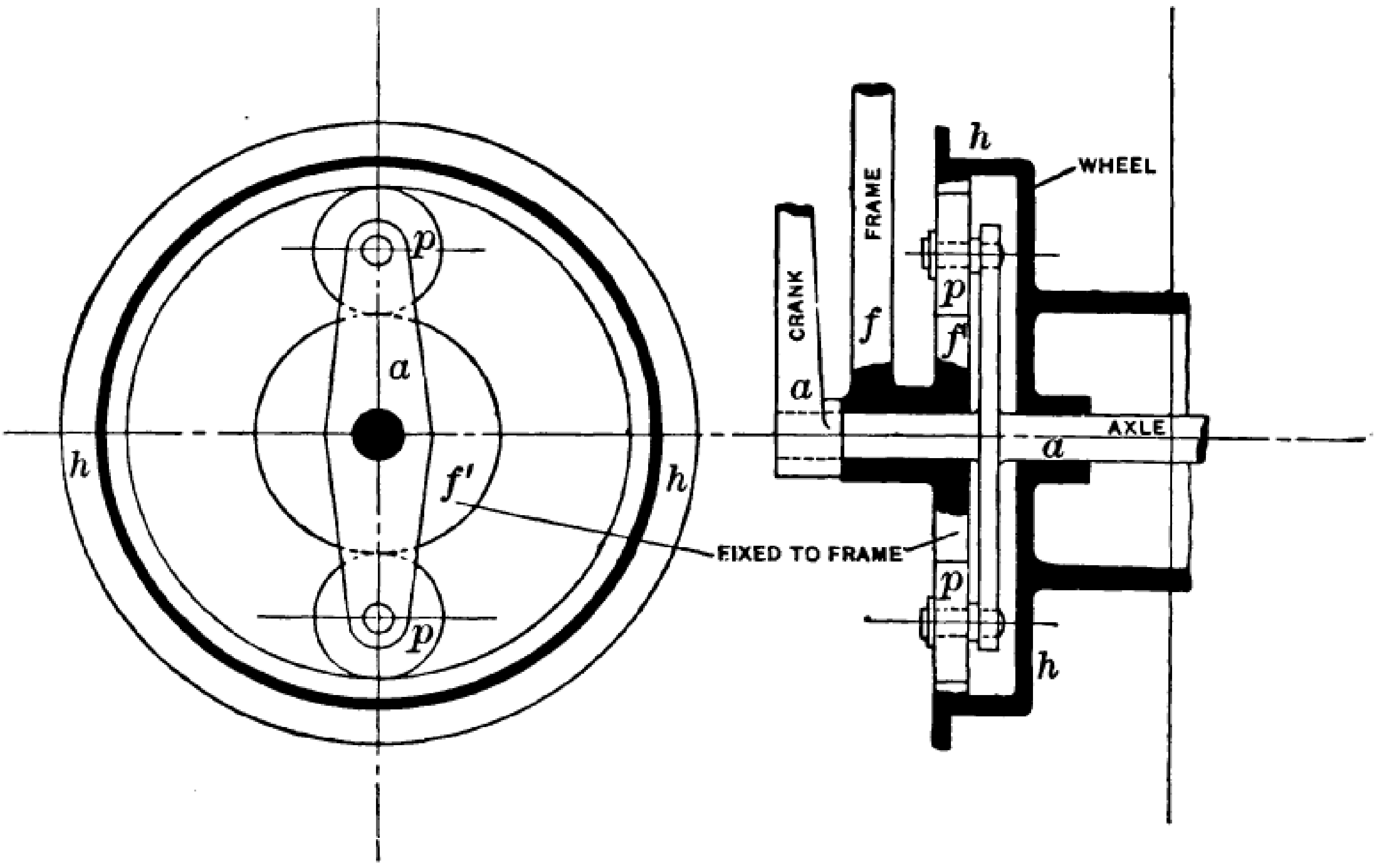

FIG. 137.

dently be arranged to give c a very slow rotary motion, say $\frac{1}{10,000}$ revolution for each revolution of the arm d.

As an example of an epicyclic reverted gear containing an annular wheel the wheel-train used in certain front-driving bicycles* may be given. In Fig. 137 ff represents

* See also the Weston triplex pulley-block described in § 78, Chapter IX.

part of the front fork of the bicycle, to which is rigidly attached the central pinion f'. The arms a, secured to the axle and cranks, carry one or more planet-wheels, p, gearing with the central pinion and with an annular wheel formed on the inside of the hub, h, of the driving-wheel. Suppose this wheel h has 60 teeth, while p has 15 and f' has 30; it is plain that $n_h = n_f + 2n_p$ if the wheels are to gear together and the wheel h is to be coaxial with f'.

Now if a were the fixed link,

$$\frac{\omega_{ha}}{\omega_{fa}} = -\frac{30}{60} = -\tfrac{1}{2};$$

therefore $$\omega_{ha} = -\tfrac{1}{2} \times \omega_{fa} = \tfrac{1}{2}\omega_{af}.$$

The ratio to be determined is the number of revolutions of the wheel h per revolution of the crank a; this is the same quantity as

$$\frac{\omega_{hf}}{\omega_{af}} = \text{velocity ratio of } h \text{ and } a.$$

Now $$\omega_{hf} = \omega_{ha} + \omega_{af}$$
$$= (\tfrac{1}{2} + 1)\omega_{af}.$$

Hence $$\frac{\omega_{hf}}{\omega_{af}} = +1.5;$$

in other words, the wheel will make $1\tfrac{1}{2}$ revolutions for each revolution of the crank, and in the same sense. A bicycle having a driving-wheel 44 inches diameter would therefore be geared to 66 inches with this arrangement.

70. Mechanisms Containing Wheel-trains.—Mechanisms are of common occurrence in which wheel trains form part of chains containing also sliding and turning pairs. Fig. 138 shows diagrammatically a "sun-and-planet" gear containing an annular wheel, forming part of a mechanism containing a slider-crank chain.

The crank a is able to rotate about the point O_{ad} with reference to a fixed frame d, and pairs with a link b, forming the connecting-rod in a slider-crank chain, of which c is the sliding block. On b, however, is formed a spur-wheel whose pitch-circle has its centre at O_{ab}. The spur-wheel gears with an annular wheel e whose pitch-circle has its centre at O_{ad}. The virtual centres are marked on the dia-

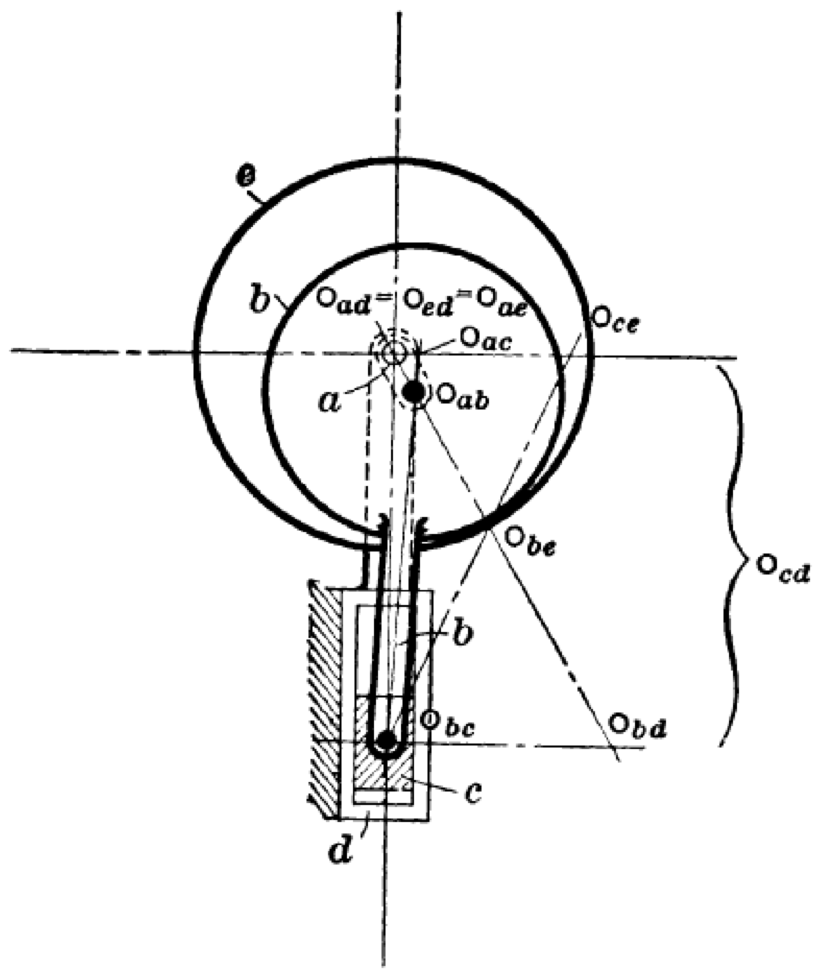

FIG. 138.

gram. We wish to find the number of revolutions of the annular wheel e for each revolution of the crank a.

As in previous examples, let N be the velocity ratio of the wheel-train; i.e., let

$$N=\frac{\omega_{ea}}{\omega_{ba}}=\frac{n_b}{n_e}=\frac{r_b}{r_e}.$$

Plainly N will be a positive fraction in this case. We note that during the action of the mechanism the average value of ω_{bd} is zero, for b simply swings to and fro, and a line marked on it describes equal angles right and left from its mid-position. Hence we may say that on the average

$$\omega_{ab}=\omega_{ad}+\omega_{db}=\omega_{ad};$$

that is, we may consider the angular velocity of a and b instead of that of a and d. The two would always be exactly equal if b always remained parallel to itself, i.e., if the connecting-rod were infinitely long.

Now
$$\omega_{ed} = \omega_{ad} + \omega_{ea}.$$

$$\frac{\omega_{ed}}{\omega_{ad}} = 1 + \frac{\omega_{ea}}{\omega_{ad}}$$

$$= 1 - \frac{\omega_{ea}}{\omega_{da}}$$

$$= 1 - \frac{\omega_{ea}}{\omega_{ba}}$$

$$= 1 - N.$$

For example, suppose e had 100 teeth while b had 95, so that $N = +0.95$; then for each revolution of the crank a, e would make $1 - 0.95 = 0.05$ revolution in the same sense. This mechanism is actually used as gearing for a capstan driven by a hydraulic engine, b being attached to the connecting-rod, while the capstan barrel is attached to e.

As another example of a mechanism containing a wheel-train we may take the wheel crank-chain of Fig. 139, which

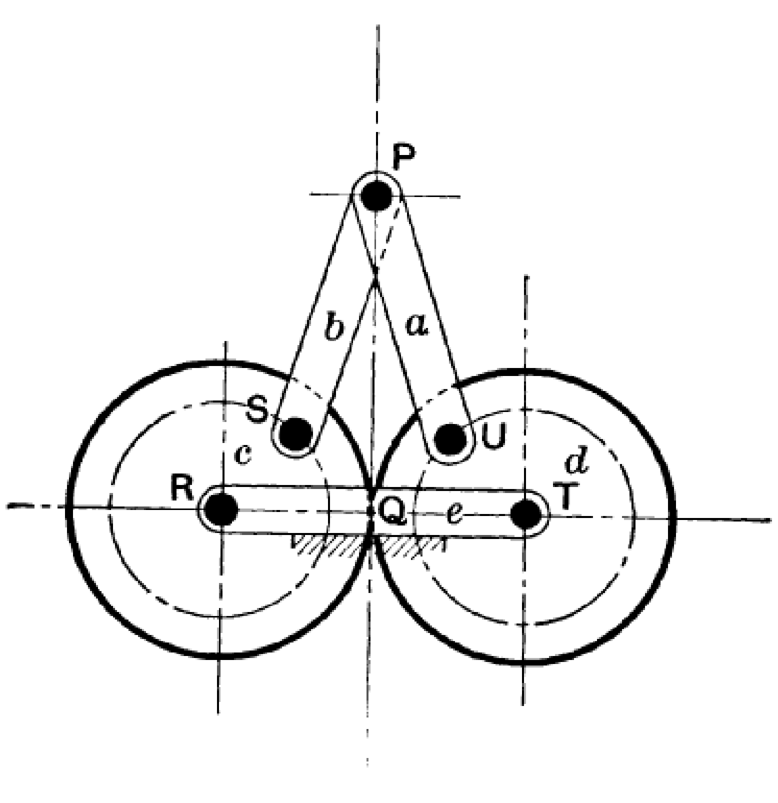

FIG. 139.

is formed by combining a simple wheel chain with an open crank-chain of five links.

If the lengths of the links a and b, and also those of c

and d, are equal, as in the figure, we obtain Cartwright's straight-line motion, in which the point P describes a straight-line path passing through Q. The purpose of the two spur-wheels is to close the five-link chain, whose motion would otherwise be unconstrained.

In Figs. 140*a* and 140*b* we have a slider-crank chain in which spur-wheels can be used for a somewhat similar purpose. Consider a slider-crank chain in which the connect-

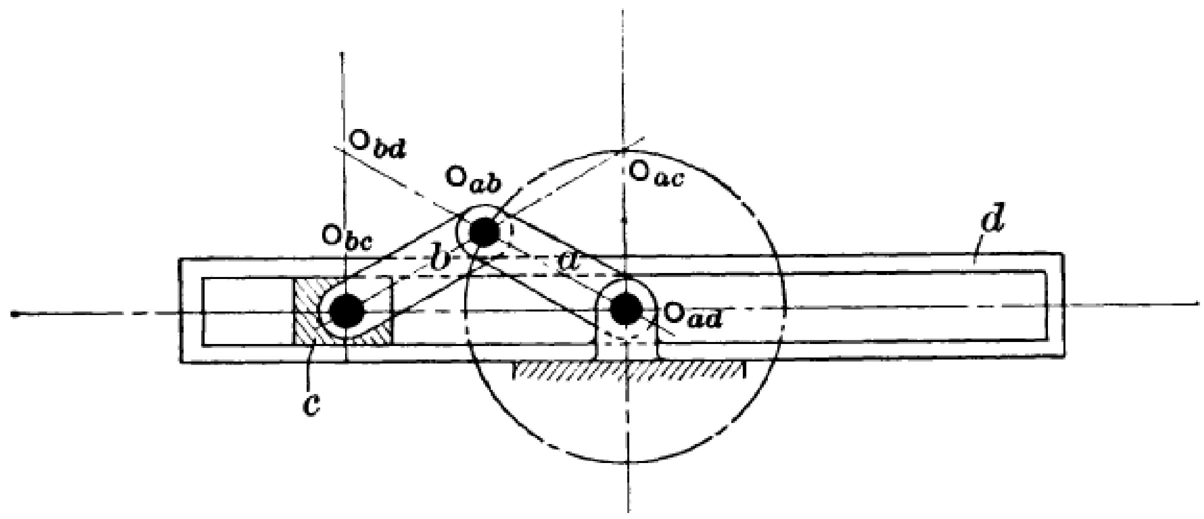

FIG. 140*a*.

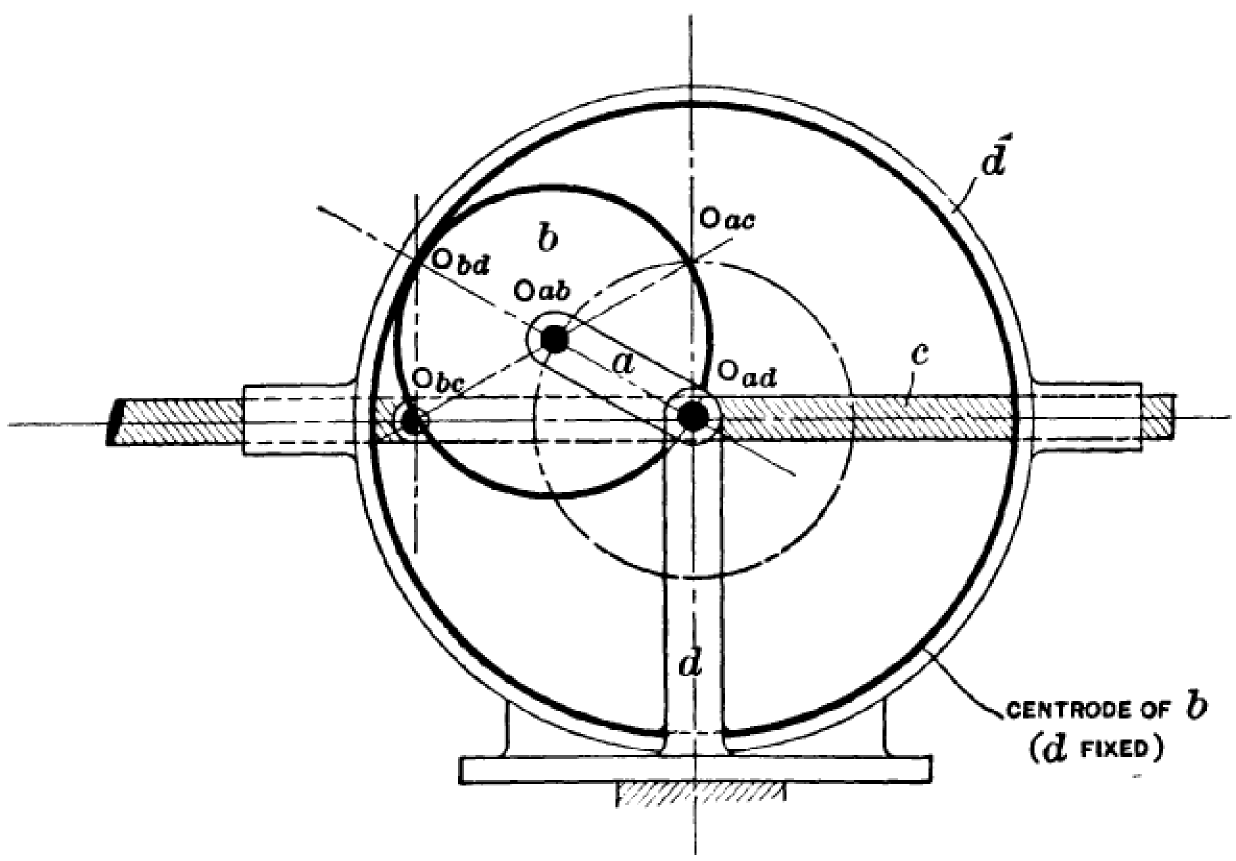

FIG. 140*b*.

ing-rod b is made equal in length to the crank a. (Compare Figs. 60 and 91.) With these proportions it is possible for the stroke of c to be (1) either twice the length of the crank, as in the ordinary slider-crank chain, or (2) four times the length of the crank, in which case O_{bc} must travel on past O_{ad}. A third possibility is that O_{bc} and O_{ad} may remain coincident, in which case *b* and *a* move together, and the mechanism will reduce to a pair of elements.

Notice in Fig. 140*a* that since the length of *b* is equal to the length of *a* and the angle $O_{bd}O_{bc}O_{ad}$ is a right angle, the points $O_{bd}O_{ad}O_{cb}$ must lie on the circumference of a circle whose radius is the length of *a* or the length of *b*. Since $O_{bd}O_{ad}$ is a diameter of this circle, it follows that O_{bd} remains always at the same distance from O_{ad}, and the centrode of *b* with regard to *d* is a larger circle whose radius is twice the length of *a*. If now (Fig. 140*b*) we attach to *d* an annular wheel whose pitch-circle is the centrode of *b* with regard to *d*, and if we fix to *b* a spur-wheel whose pitch-circle is the centrode of *d* with regard to *b*, these wheels will gear together, and will compel O_{bd} to remain always at a fixed distance from O_{ad}. If these wheels were not provided we should have a change-point at the instant when O_{bc} passes O_{ad}, but if the virtual centre O_{bd} is compelled to remain at a fixed distance from O_{ad} by the action of the spur-wheels, O_{bc} is compelled to continue its travel, and the mechanism is not permitted to change. Obviously this arrangement is really a case of pair-closure at a change-point. (Compare the examples in § 59.) The only really essential portions of the wheels are therefore those teeth which are in gear while O_{bc} is passing O_{ad}.

71. Cam-trains.—The name cam-train is applied to mechanisms containing a rotating disc (generally non-circular) or a sliding plate, the profile of which forms one element of a higher pair and gives some desired periodic motion to the second element of the pair. Such a cam-pair may be closed by forming one element into the geometrical envelope of all possible positions of the other element. Mechanically cam-pairs usually possess the disadvantage of small wearing surface and rapid wear, common in higher pairs. Almost invariably force-closure is necessary to make up for the looseness of fit following on wear. A cam-train is in general a mechanism of three links; such, for example, is the cam-train found in the stamp-mill used for crushing hard ores (Fig. 141).

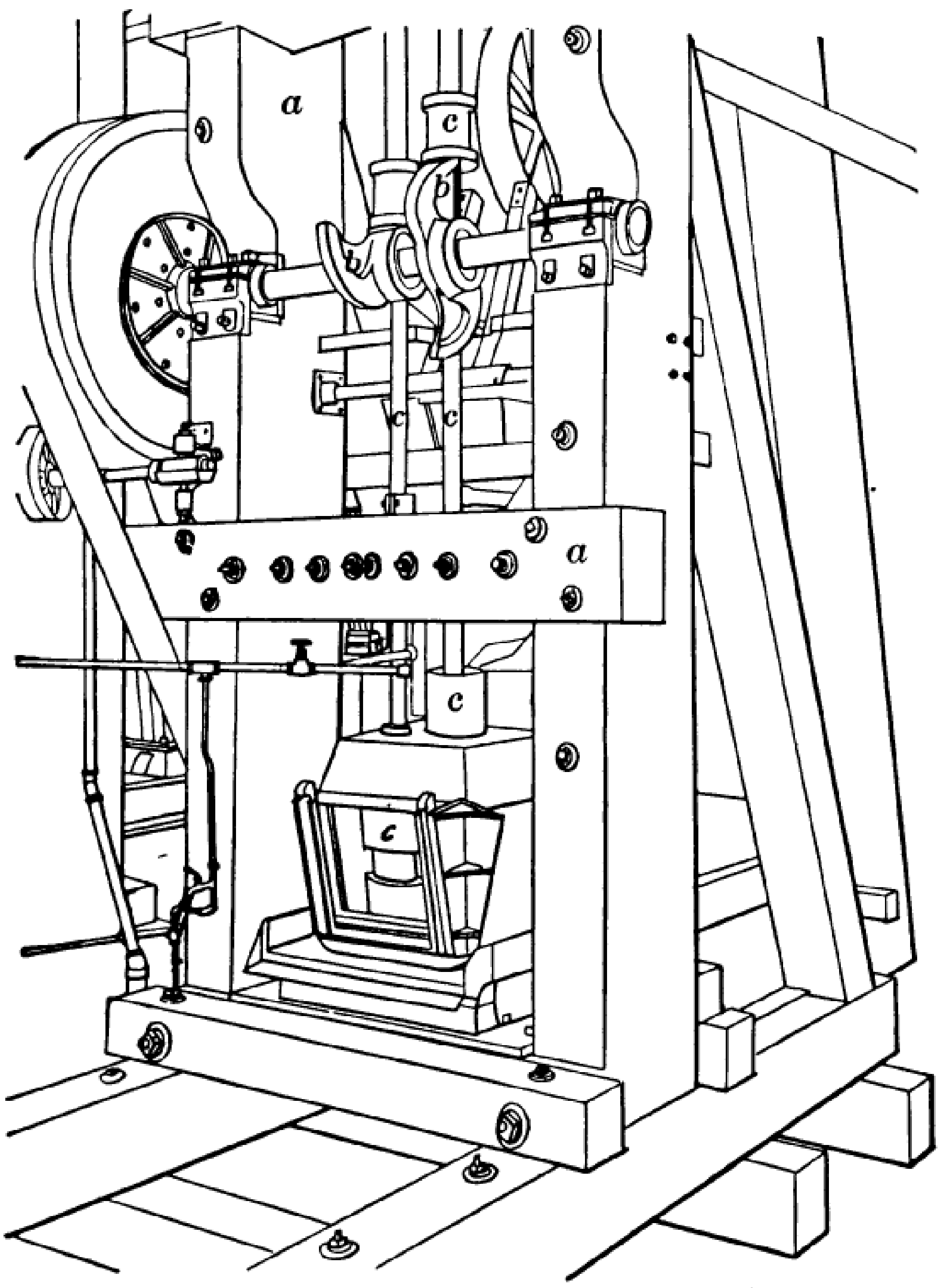

FIG. 141.

A rotating shaft carries the cams *bb*. These successively lift and let fall the stamps *cc*, which are guided by means of the framework *a*. It will be noted that the cam-pair *bc* is force-closed by the weight of the stamp itself, and also that the form of the cam is such that in any position during the upward stroke it is touched by the horizontal under surface of the collar on the stamp-rod. This fact has to be consid-