CHAPTER III.

PLANE MECHANISMS CONTAINING ONLY TURNING PAIRS.

26. Quadric Crank-chains.—If we endeavor to make a plane mechanism out of links containing only turning pairs, we find that the least number of links with which this can be done is four. A chain of *three* links so connected forms

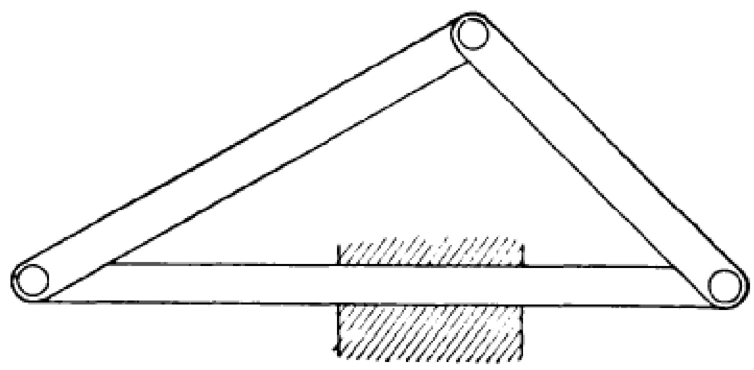

FIG. 39.

an arrangement which is of value as a *structure* (a simple triangular roof-truss), but is of no service as a mechanism, since its parts can have no relative motion.

On the other hand, a simple chain of *five* or any greater number of links connected by turning pairs is equally useless as a mechanism, since the relative motion of at least two of its links is not constrained, as has been shown in § 3.

Let us consider, therefore, a chain of four links connected by turning pairs whose axes are parallel. When the links of this chain are of unequal lengths the smallest is called the crank, and since the four links form a quadrilateral, the chain has been called by Reuleaux * the *quadric* (*cylindric*) *crank-chain*. The term 'cylindric' distinguishes this chain

* Kinematics of Mach., §§ 62–65.

from the corresponding spheric chain, in which the axes are not parallel.

In quadric crank-chains it will be convenient to distinguish between links having a swinging or partial turning movement and those which can execute complete rotations relatively to the fixed link in the chain.

The former links will be called levers, the latter cranks. It is obvious that by altering the relative lengths of the links we can obtain different relative motions, and hence different mechanisms. From these, again, other different mechanisms are produced by inversion of the chain.

27. Virtual Centres and Centrodes.—Let *abcd*, Fig. 40, represent the four links of a quadric crank-chain. Each of these links will have motion relatively to every other, and hence we shall have six virtual centres. Four of these centres are readily identified as the axes of the turning pairs; for instance, the virtual centre of *c* with regard to *d*, or of *d* with regard to *c*, is obviously the point 3, and may be

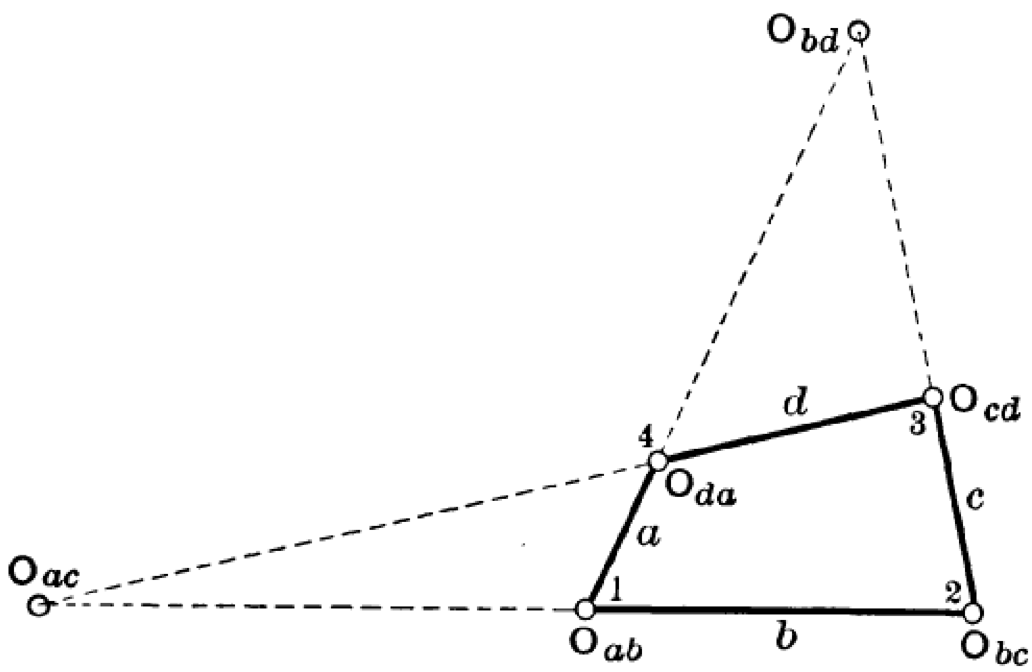

FIG. 40.

indicated as O_{cd} or O_{dc}. In the same way we have O_{ad}, O_{ab}, and O_{ac}; all these points are in fact permanent centres as regards their own pair of links. Remembering that for any three bodies having plane motion the three virtual centres lie in one straight line, it is easy to see that O_{ac} must lie at the join of the straight lines drawn through O_{ab} and O_{bc}, and through O_{ad} and O_{cd}. In the same way O_{bd} is at the intersection of the lines $O_{ab}O_{ad}$ and $O_{cb}O_{cd}$.

Supposing b to form the *frame* or fixed link, it is seen that since O_{ab} and O_{bc} are permanent centres, the centrodes of a and c with regard to b are points, namely O_{ab} and O_{bc}.

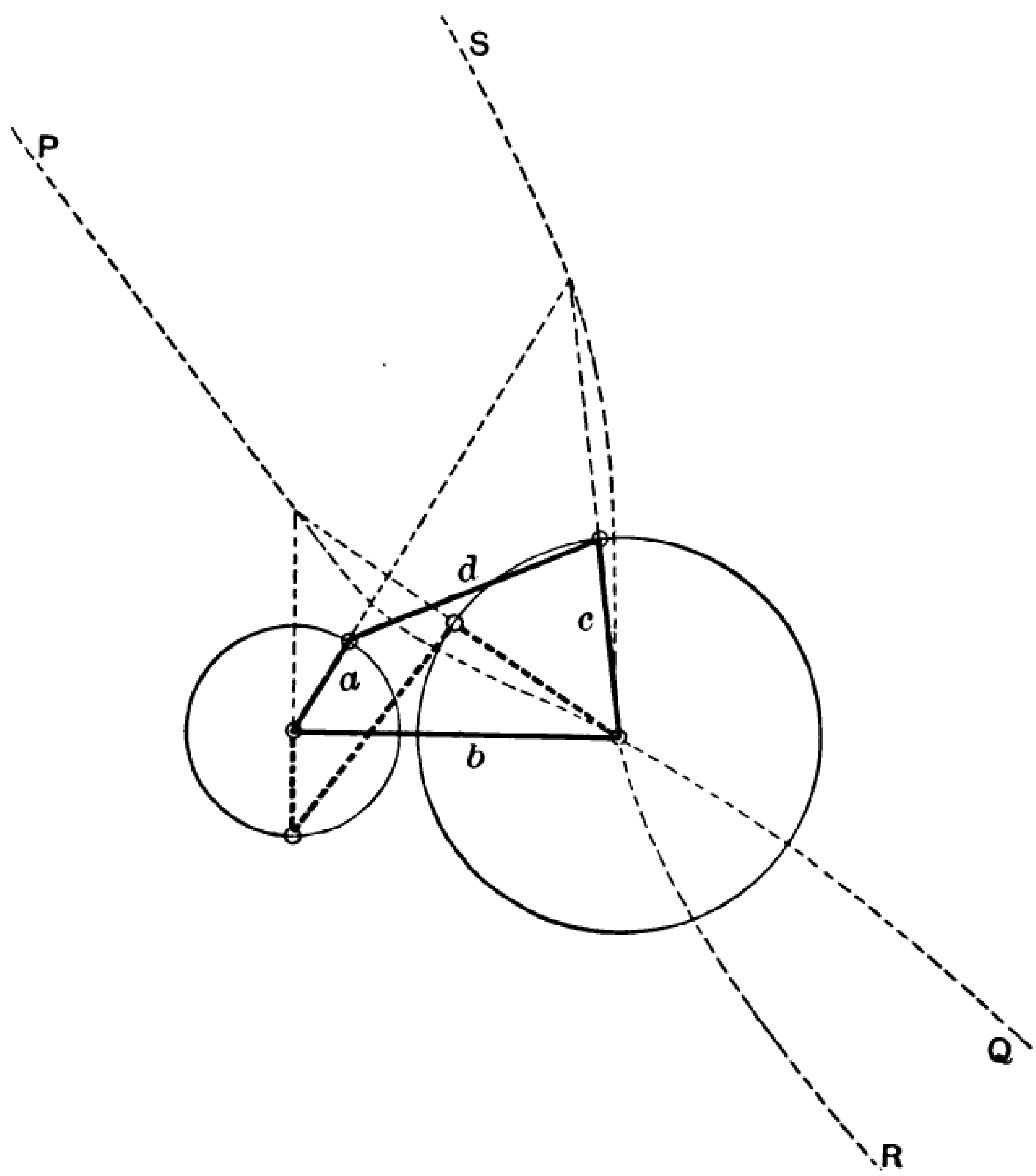

FIG. 41.

The centrode of d with regard to b is the locus of O_{db} and takes the form of a curve having four infinitely distant points; portions of it are readily drawn by finding a series of positions of O_{db} corresponding to successive positions taken up by the three links a, c, and d. In a similar way may be obtained the centrode of b with regard to d (supposing d to be the fixed link). The curve $PQRS$

in Fig. 41 represents the centrode of d with regard to b; the construction for two points on the curve is shown.

28. Angular Velocities.—It is frequently of importance, having given the angular velocity, say, of the link a, to find

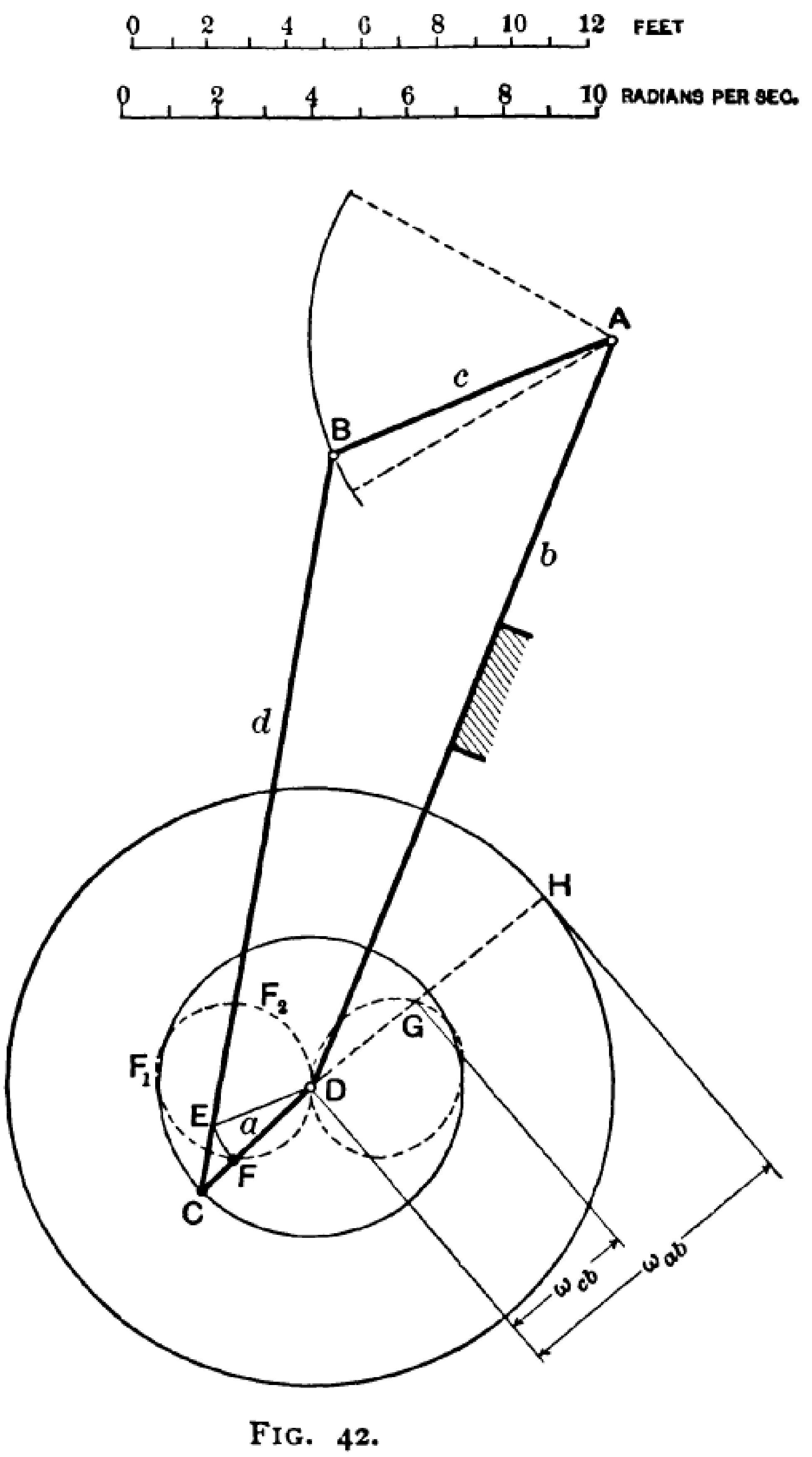

FIG. 42.

that of any other link, say c, or, in other words, to determine the angular velocity ratio of the chain. This can be very simply done by construction.

In Fig. 42 let $ABCD$ represent the mechanism, AD being the fixed link, and the uniform angular velocity of CD being known. It is required to determine the angular velocity of AB for any position of the mechanism.

Draw DE parallel to AB, and cutting BC, or BC produced, in E. With centre D and radius DE mark off DF

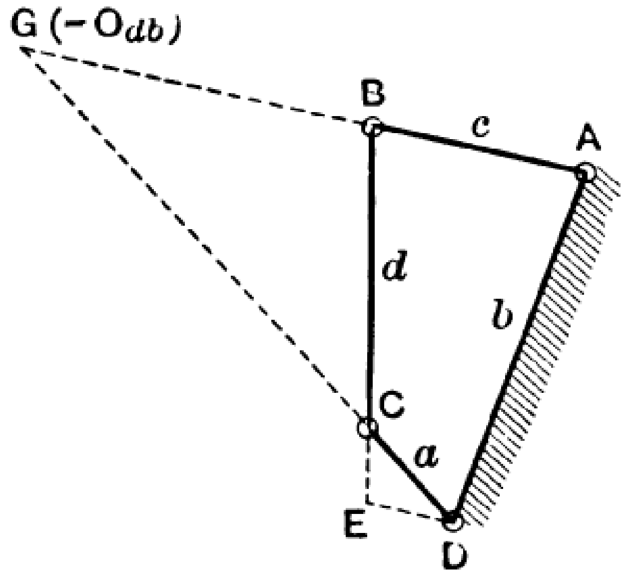

FIG. 43.

along DC. Then DF represents the angular velocity of c on the same scale as that on which AB represents the angular velocity of a, and if a series of points such as F be obtained, the curve FF_1F_2G . . . drawn through them will form a polar diagram of angular velocities for c and a.

To prove this construction, let ω_{cb}, ω_{ab} be the angular velocities of c and a respectively with regard to b. In Fig. 43 find O_{db}, the intersection of AB and DC at G, and draw DE parallel to AB, meeting BC in E.

Since the link d is turning for the instant about G, we must have

$$\frac{\text{linear velocity of } B}{\text{linear velocity of } C}=\frac{GB}{GC}.$$

Now $$\omega_{cb}=\frac{\text{linear velocity of } B}{AB},$$

and $$\omega_{ab}=\frac{\text{linear velocity of } C}{CD};$$

hence $$\frac{\omega_{cb}}{\omega_{ab}}=\frac{\text{linear velocity of } B}{\text{linear velocity of } C}\cdot\frac{CD}{AB}$$

$$=\frac{CD.GB}{AB.GC}.$$

But by construction the triangle BGC is similar to the triangle EDC; hence

$$\frac{GB}{GC}=\frac{ED}{DC}$$

Therefore $$\frac{\omega_{cb}}{\omega_{ab}}=\frac{CD.ED}{AB.DC}=\frac{ED}{AB}$$

Thus if AB represents the angular velocity of a with regard to b, ED represents on the same scale that of c with regard to b.

Fig. 42 gives such a velocity diagram, drawn to scale, for the beam of a beam-engine when the crank rotates uniformly. For comparison the circle of radius $DH=AB$ has been drawn, so that for any radius DGH the intercept DG represents ω_{cb}, just as DH represents ω_{ab}. The polar curve of velocity is shown by a dotted line.

The distances taken are:

$$AB=8 \text{ feet}=DH;$$
$$BC=20 \text{ feet};$$
$$CD=4 \text{ feet};$$
$$DA=21.5 \text{ feet}.$$

When the crank is in the position DH the angular velocity ratio is

$$\frac{DG}{DH}=\frac{3.5}{8}=0.438,$$

or at that particular instant the beam is swinging with 0.438 the angular velocity of the crank. If the crank rotates uniformly at 60 revolutions per minute or 6.28 radians per second, in the position AB the beam is moving with an angular velocity of $6.28 \times 0.438 = 2.75$ radians per second.

From the curve of angular velocity thus obtained we might draw the curve of angular acceleration by the construction described in § 22. Notice that the construction just described can still be applied in positions of the mechanism where O_{bd} is inaccessible, i.e., when AB and CD are nearly parallel, and when the relative angular velocities, therefore, could not be found from the position of the virtual centres.

29. Inversions of the Quadric Crank-chain.—In the particular example of the quadric crank-chain just examined, the lengths of the links are such that while the link a executes complete rotations with reference to b or d, c only swings. a is then a crank, c a lever, and if the link b is the fixed one, the resulting mechanism is called the lever crank-chain.

In order that a may execute complete rotations with regard to b it is necessary that $a+b \leqq c+d$, while also $a+d \leqq c+b$, a being the smallest of the links.

With these proportions let us see the result of inversion of the chain. On considering the relative motions of the links we find that the motion of a relatively to b or d is that of complete rotation, while with regard to either of the

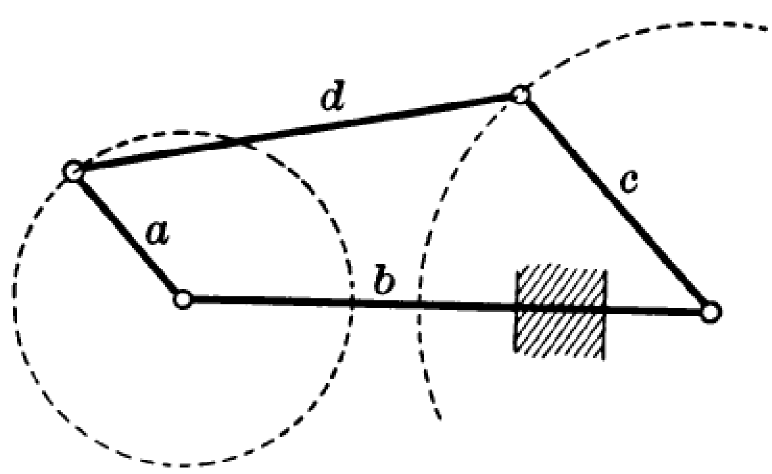

FIG. 44.

same links c only swings or performs partial revolutions. As has been already pointed out, inversion can make no change in the *relative motions* of the links, and hence the mechanism will remain a *lever crank-chain* whether *b* or *d* be the frame (or fixed link).

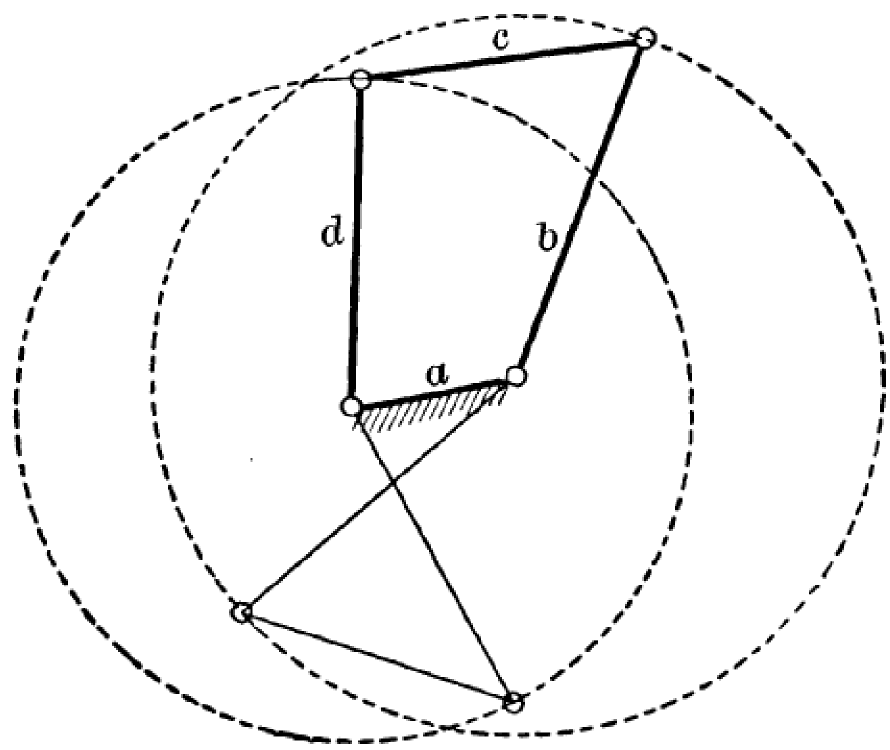

FIG. 45.

On fixing *a*, however, as in Fig. 45, a new mechanism is obtained which may be called the *double crank*, inasmuch as both *b* and *d* can now execute complete rotations about the axes of the pairs *ba* and *da*.

This mechanism is used in practice as a drag-link coupling, *b* and *d* being represented by the discs keyed on to the two shafts, *a* by the frame containing the bearings, and *c* by the drag-link connecting the pin on *d* with that on *b*. The mechanism is also employed in the construction of feathering paddle-wheels.

When used for this purpose the object is to cause the floats to enter and leave the water edgewise, (so as to avoid splashing,) while remaining vertical at the bottom of their travel.

Thus suppose *A*, *B*, *C*, Fig. 46, to be points on the path of the outer edge of a float, *AC* being the water-line. The steamer has a certain speed relatively to the water, so that a point *A* on the wheel is moving horizontally with a velocity

AD in common with every other point on the vessel, the length of *AD* thus representing the speed of the ship to any convenient scale. In virtue of the rotation of the wheel, *A* has also a linear velocity (represented by *AE* to the same scale), relatively to the ship; therefore the real direction in which

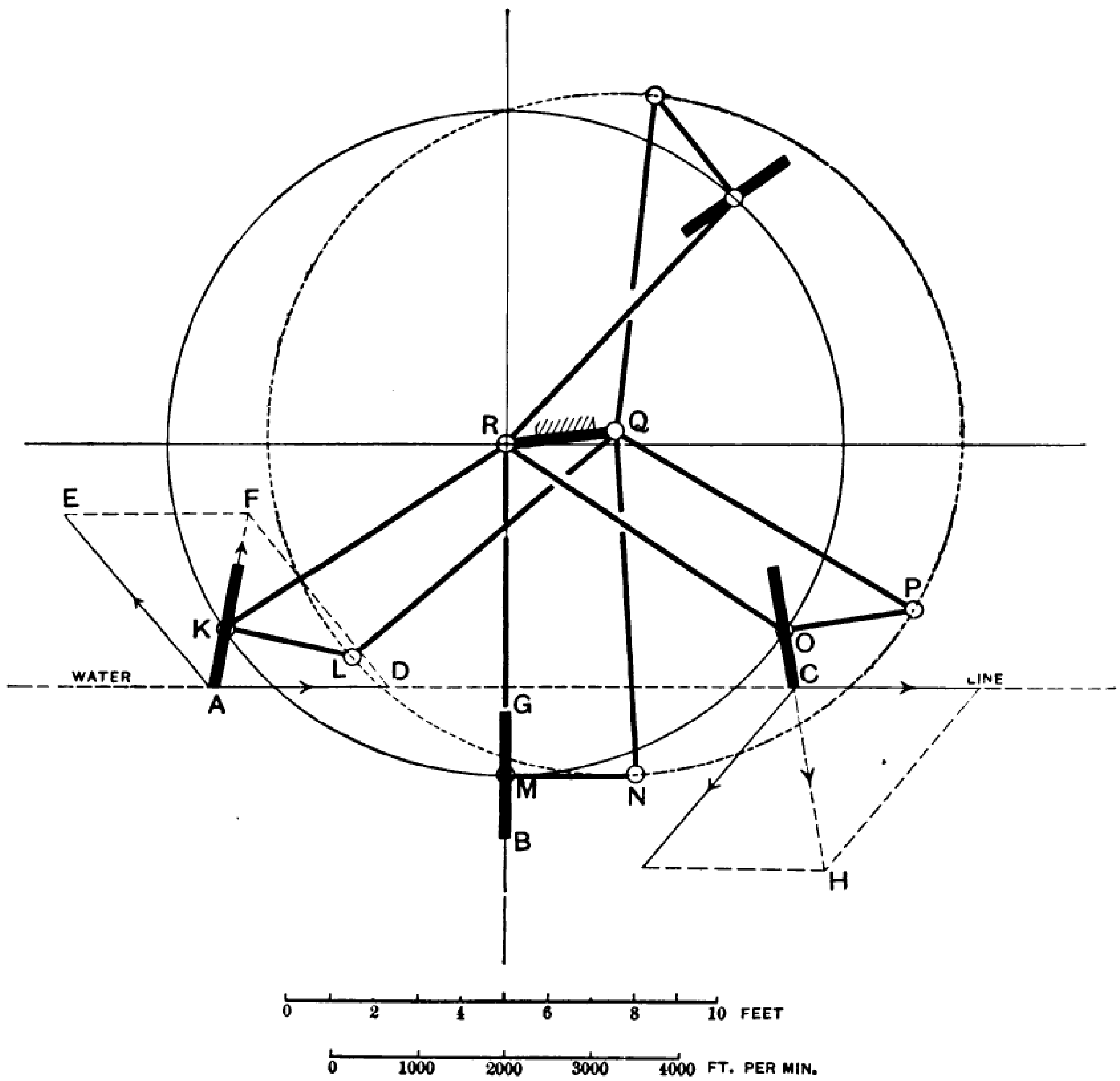

FIG. 46.

A moves relatively to the water is *AF*, the diagonal of the parallelogram *AEFD*. The floats then at entrance and exit should lie in the positions *AF* and *CH*, while the float in its lowest position should evidently be vertical, as at *BG*. The floats are pivoted at their centres to the framework of the wheel and have attached float-levers KL, MN, OP. The ends L, N, and P are all connected by radius-rods to an eccentric-pin, generally fixed on the sponson-beam of the paddle-box. The centre of this pin is of course at Q, the centre of the circle passing through N, P, and L, while R is the centre of the wheel itself. It will be seen that the paddle-wheel arm, the float-lever, the radius-rod, and part of the ship's structure form a double-crank mechanism, thus giving the floats the desired movement. Fig. 46 is drawn to scale from the following data:

Speed of ship 21 knots = 2026 feet per minute.
Diameter of wheel (ext.) 18.5 feet.
Revolutions per minute 48.
Breadth of floats 3′ 0″.
Immersion of lower edge 3′ 6″.
Length of float-levers 3′ 0″.
Speed of outer edge of float 2790 feet per minute.

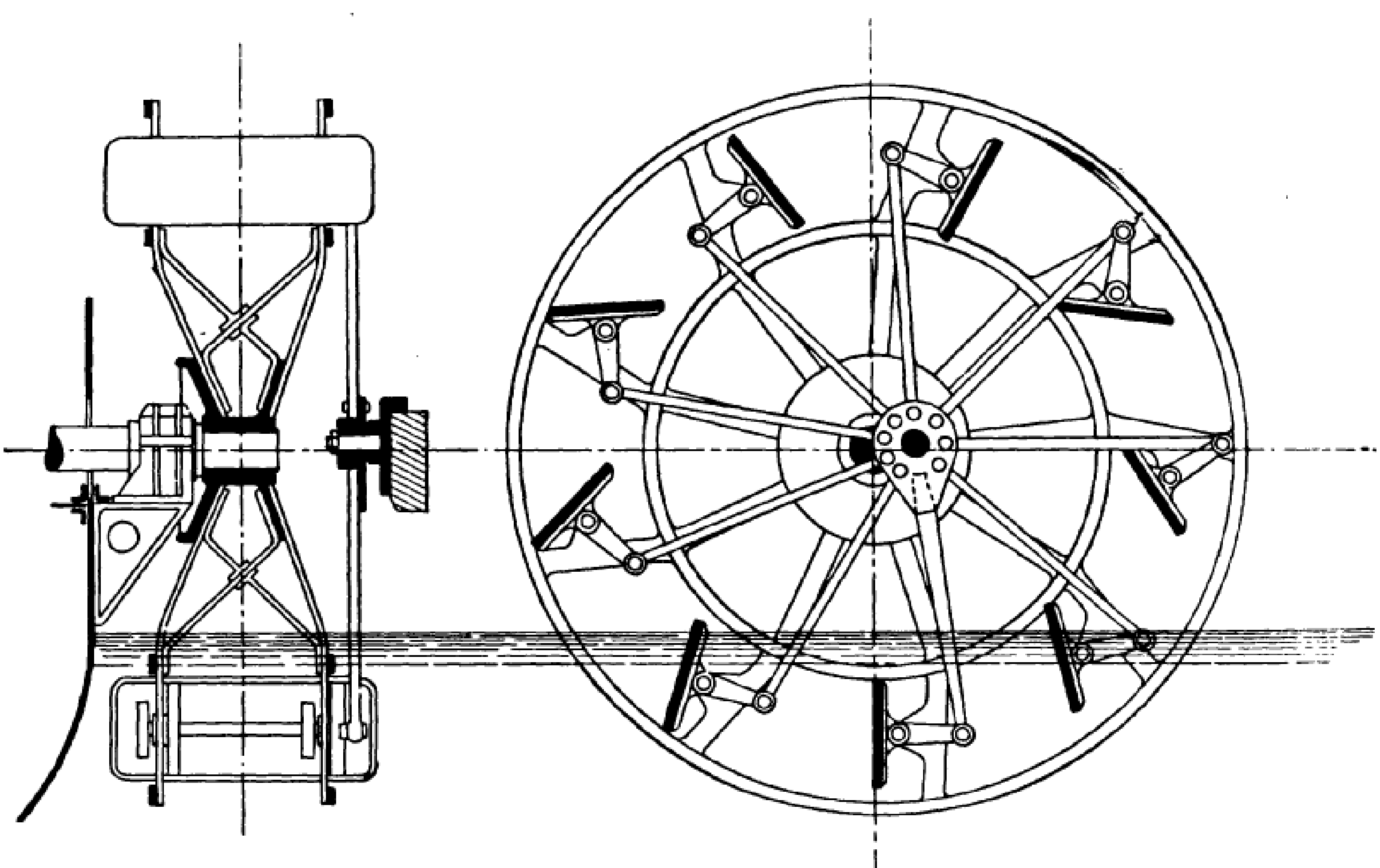

FIG. 46*a*.

Fig. 46*a* is a drawing of the arrangement of an actual feathering paddle-wheel. The reader will have no difficulty in recognizing various links in the double-crank mechanism.

Next suppose c (Fig. 44) to be the fixed link. Remembering that the relative motions of b and c and d and c are partial and not complete rotations, we see that a third mechanism, the *double lever*, is the result of this inversion.

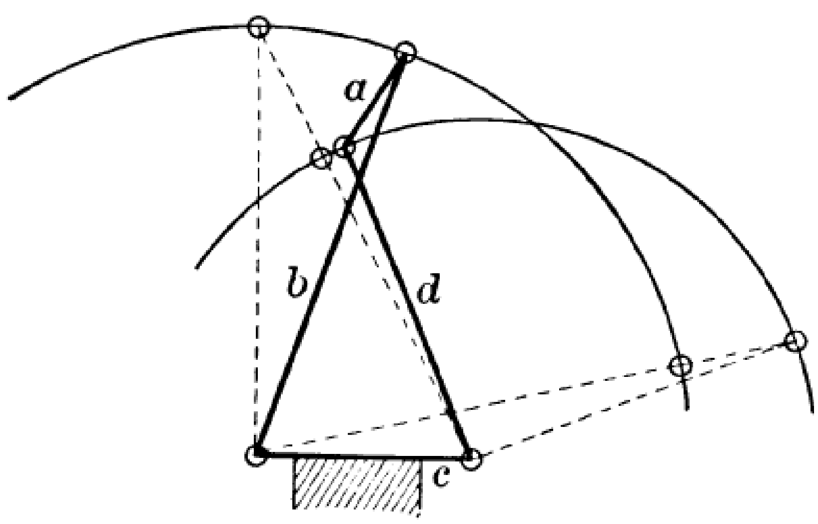

FIG. 47.

The double-lever mechanism is shown in Fig. 47, and such an arrangement finds an application in certain approximate straight-line motions (compare Fig. 56b).

The result of the various inversions of the quadric crank-chain may be summarized as follows:

Fixed Link.	Mechanism.
a	Double crank.
b	Lever-crank.
c	Double lever.
d	Lever-crank.

The four inversions have thus given us three different mechanisms.

Certain special cases of the quadric crank-chain have peculiarities which are of interest. Suppose that the lengths of the links are such that either $a+b=c+d$ or $a+d=c+b$, b being the fixed link. This condition is shown in Fig. 48.

As we have seen, it is still possible for a to execute complete rotations, but it will now be found that c can swing on either side of the fixed link *b*. We obtain one position of the chain in which all the links are in a line, and the angle through which *c* can swing is then doubled. This condition is expressed by saying that the mechanism passes through

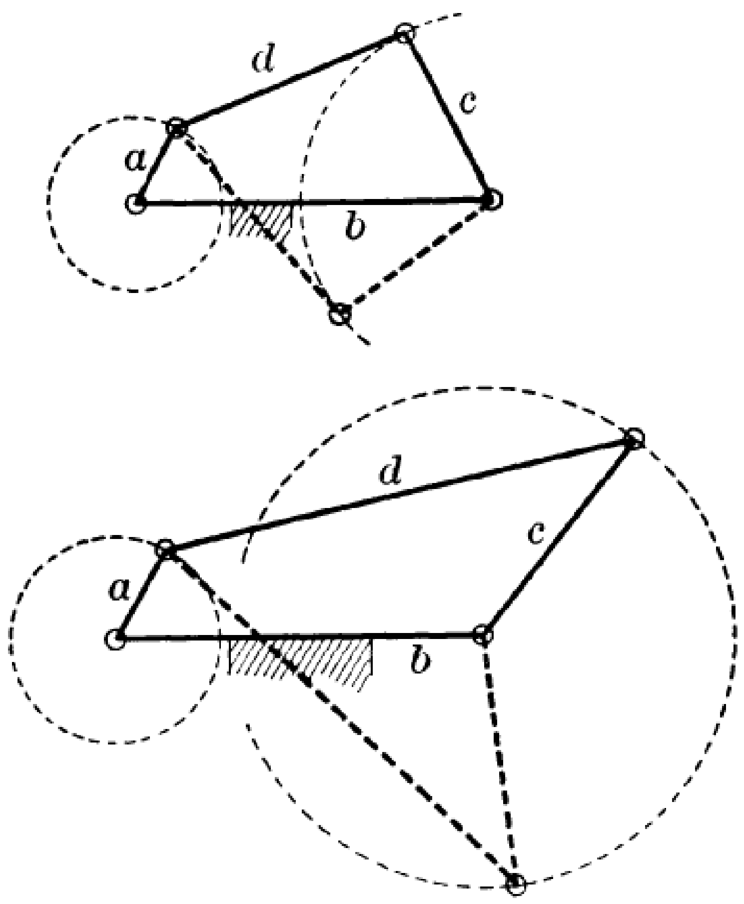

FIG. 48.

a *change-point* when all the links are in line, and for any given position of the link *a* the mechanism can assume either the position shown by the full lines or that shown by the dotted lines in the figure.

30. Change-points and Dead-points. — A *change-point* may be defined as a position of a mechanism in which such a want of constrainment occurs that it is possible for the arrangement to transform itself into another mechanism, or, in some cases, into a pair of elements.

A very familiar instance of such a change-point occurs in the quadric crank-chain when of the form known as *parallel cranks;* that is, when *a* and *c* are equal in length. and considerably shorter than *b* and *d*, which are also equal This is of course a particular case of the condition illustrated in Fig. 48, and is shown in Fig. 49.

At the instant when all the links are in a line it becomes possible for the mechanism *abcd* either to take up such a form as *abc′d′* or to continue its motion in its original form. The mechanism in question is of common occurrence in locomotive engines having coupled driving-wheels, and the necessary constraint at the change-points is provided by duplicating the chain, namely, by arranging another pair of cranks and a coupling-rod on the other side of the engine, so placed that the change-points of the two chains do not occur at the same time. Other methods of obtaining a similar object will be found discussed in a later chapter.

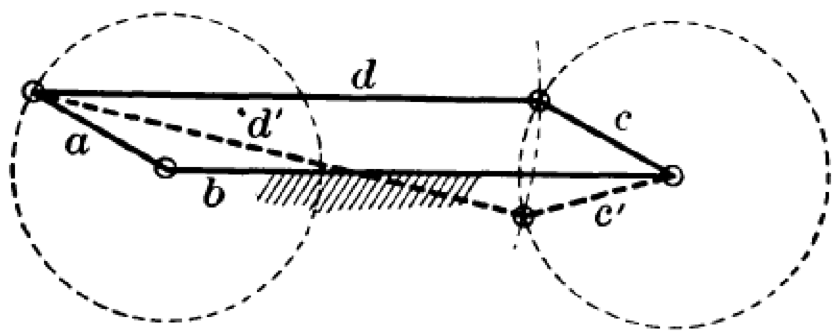

FIG. 49.

By the term *dead-point* in a mechanism is meant a position of the various links such that one of them directly opposes itself to the action of the forces tending to produce motion. The term was first applied by Watt to those positions of the crank and connecting-rod in a steam-engine in which the axes of three turning pairs lie in one plane, so that a force applied to the piston is not able to cause any torque on the shaft. It is plain that, in the absence of some means of overcoming this difficulty, the further motion of the chain becomes impossible, and the chain may be regarded as incomplete. The occurrence of dead-points must not be confused with that of change-points, although they may, and often do, occur together.

It is important to note an essential difference between dead-points and change-points. The occurrence of a dead-point depends on the particular link to which the driving force is applied, and on the manner of its application. For instance, in the lever-crank mechanism of Fig. 42, if the crank be turned by the application of a continuous torque to its shaft, no dead-point exists. In the very same mechanism, however, if the driving effort be applied to the lever c (as in a beam-engine), dead-points occur twice in each revolution of the crank.

A change-point, on the other hand, is caused by the configuration of the chain itself, and is present whichever link is fixed, so long as the chain is the same.

31. Special Forms of Quadric Crank-chain.—Suppose in the quadric crank-chain that we make $a = b$ and $c = d$; we then obtain a chain called by Professor Sylvester the *kite*, from its form. When one of the links is fixed, and the motion examined, it will be found that there are two change-

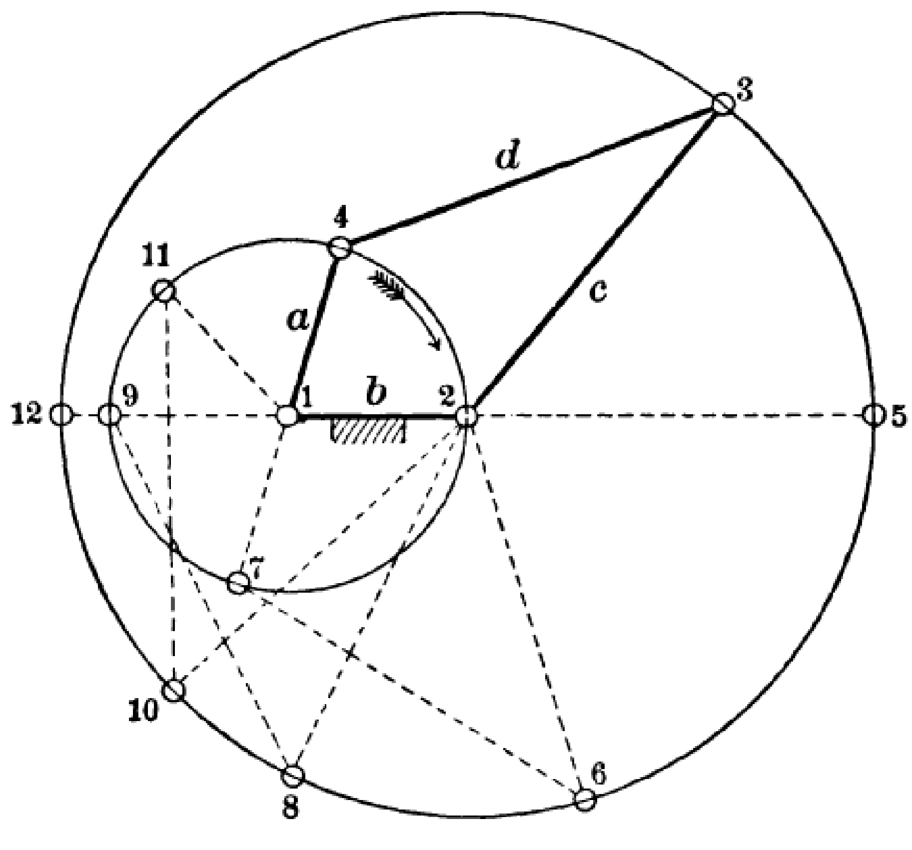

FIG. 50.

points. Thus, in Fig. 50, imagine that a rotates in the direction of the arrow, b being fixed. When the joint 4 coincides with 2 the chain becomes, for the instant, a turning pair, having its centre at 2, c and d rotating together. If a continues its motion for another complete rotation, another change-point occurs, the chain having passed through the